STATIC STRESS ANALYSIS OF A SAILPLANE WING CONSTRUCTED OF COMPOSITE MATERIALS

By

Ronald D Kriz

California Polytechnic State University
San Luis Obispo

1973

EXTRAORDINARILY THOROUGH JOB.!

PROJECT: A
REPORT: A

Submitted: Dec 3, 1973

Project Advisor: ______

Grade: ______________

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. SUMMARY

In this paper an analytic and experimental static stress analysis of a standard class Libelle sailplane wing is outlined. The analytic static stress analysis is compiled into a computer program which idealizes the wing as a tapered, multicelled, thin-walled, nonsymmetric, cylinder. Of particular interest was the comparison of analytic and experimental shear stress distributions under a static load. Unfortunately only the experimental shear stress distribution was obtained. The comparison of analytic and experimental shear distribution is pending the debugging of the computer program. A favorable comparison would indicate a representative shear center location for a real wing.

II. INTRODUCTION

High performance sailplanes are constructed almost entirely of composite materials. The wings of an open class Libelle sailplane are constructed of balsa wood, epoxy resins, and glass fibers as shown in Figure 1. Amazingly no metal is used to resist bending or torsion loads in the wing. The entire wing span has no ribs but only a spar cap to resist bending and a thick laminate skin to resist some bending but mostly torsion and shear loads. The thick laminate fiberglass wing gives an aerodynamic advantage over other types of construction.

Sailplanes are out of necessity designed aerodynamically clean. During construction the desired wing geometry can be closely controlled by accurately shaped molds. This is a necessary condition for laminar airfoils. The smooth surface finish (no rivets) also helps contibute to laminar flow and decreases drag. The thick laminate skins do not buckle (oil can) under large bending loads and can therefore maintain the critical laminar airfoil crossectional geometry under load. Unfortunately the structual evaluation of composite materials is not an easy task and has been the topic of much research work. The author believes that it is for this reason the trend in american sailplane design is today still all metal.

The objective of this project was to develope analytic methods representative of a real sailplane wing constructed of composite materials.

The author was fortunate to obtain a twelve foot section of an open class Libelle sailplane wing. With this wing analytic and experimental methods were developed for a static stress analysis.

III. PROCEDURE

The Libelle wing was mounted as a cantilever beam as shown in Figure 2. The wing was loaded by applying a uniform distributed load along the trailing edge of the wing. This sumulated a lift distribution of 1.027432 lb/in and a torque distribution of 1.027432(14.25-0.0639 x span) in-lb/in. Strains were recorded from rosettes located around the wing at a section 18 inches from the root, Table I.

Sections of the wing were removed and tensile tested for materials properties, as shown in appendix B, so that stress-strain relationships could be used to experimentally determine the shear stress distribution at a particular airfoil crossectiion.

An analytic static stress analysis was programmed in fortran IV. An example problem was used in debugging and checking accuracy of calculations in the computer program, appendix A. Once the program is debugged the geometry and material properties for the Libelle wing will be inputted and the computer program will output the shear stress distribution among other results. The objective is to then compare the experimental with the analytic shear stress distribution.

IV. THE STRESS ANALYSIS OF A TAPERED, MULTICELLED, THIN-WALLED, NONSYMMETRIC CYLINDER

The computer program idealizes the wing as a tapered, multicelled, thin-walled, nonsymmetric cylinder. The wing geometry is defined at the root and tip airfoil crossections as shown in Figure 6.

The use of the balsa core sandwich is to resist wrinkling of the thin skin under compressive loads and was assumed ineffective in resisting torsion or bending loads. This restriction was necessary since a relatively simple mathematical model was desired for programing.

COULD BE ACCOUNTED FOR BY USING MODULUS – WEIGHTED THICKNESS. (MINOR)

The following is a simplified step by step procedure used by the computer program to statically stress analysis the idealized wing.

I) Assume a lift, drag, and torque distribution as a function of span position.

  A) Assume a lift distribution L=f(y)
  B) Assume a drag distribution D=f(y)
  C) Assume a torque distribution T=f(y)

<u>Comments</u> The lift, drag, and torque distributions represent the loads acting on the wing at the aerodynamic center.

II) Calculate direction cosines for aerodynamic center line.

III) Establish a set of reference axis for defining an airfoil shape at any span position.

  A) Define the location and shape of the root and tip airfoils using the reference axis.

  B) Calculate any other airfoil shape by using analytical geometry.

Comments The airfoils shape was approximated by drawing line segments between N points defining the location of the thin skin. The wing shape is defined by drawing straight lines between corresponding points defining the root and tip airfoil shapes.

DOES NOT EXACTLY ACCOUNT FOR AIRFOIL SHAPE VARIATION? (MINOR)

IV) At any spanwise airfoil section

  A) Determine the location of the aerodynamic center
    1) Define the location of the root and tip aerodynamic center.
    2) Calculate any other aerodynamic center by using analytic geometry.

Comments Any other aerodynamic center location was defined along a line drawn between the root and tip aerodynamic center.

B) Determine the centroid location and moments fo inertia about a set of centroidal axis referenced parallel with the reference axis.

Comments The accuracy of the centroid location and moments of inertia depend on the skin thickness. The thinner the skin the more accurate the results. ✓

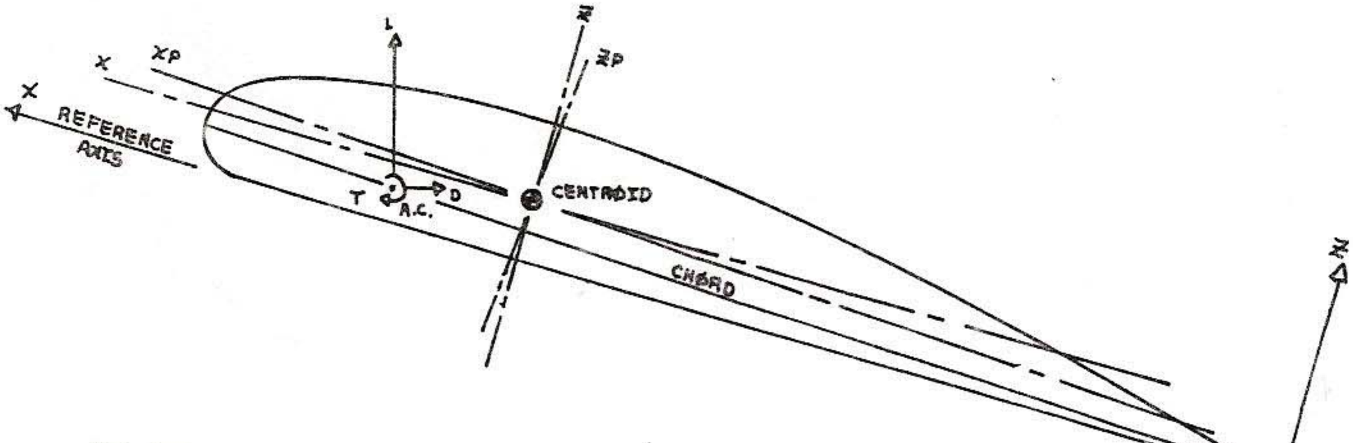
C) Determine the location of the principal centroidal axis and the moments of inertia.

Comments The angle of rotation needed to locate the principal axis is calculated by using the moments of inertia calculated from the previous centroidal axis. This assumed principal axis is then used in calculating the product of inertia. If this product of inertia is not with in a set minimum value then the axis is rotated again until this minimum value is obtianed.

SEEMS TO ME AS THOUGH YOU COULD FIND THE PRINCIPAL AXES DIRECTLY, WITHOUT HAVING TO ITERATE.

D) Determine the bending moment, shear force, and torque.
  1) Assume the lift, drag, and torque act at the aerodynamic center.
  2) Use the trapezoidal numerical method for calculating the torque and shear forces.
  3) Use a numerical method analogous to the trapezoidal for calculating the bending moments.
  4) Using the direction cosines previously calculated transfer the loads from the aerodynamic center line to a set of axis parallel to the free stream velocity.
  5) Using the angle of attack and chord angle the loads along the X,Z,Y axis and the XP,ZP,Y axis are calculated.

Comments The program was generalized to handle any shaped load distribution. But interval spacing for elliptic shapes gives the most accurate results. ?



E) Determinethe shear center location and the shear flow due to the shear forces acting along the principal centroidal axis and at the shear center.

Comments The method of closed thin walled sections was used.[5] The accuracy depends on how many points are used to define the shape of the thinwalled crossection and the skin thickness. An error in shear flow values exists because the the formulas assumed nontapered beams and the shear flows were averaged and not curve fitted between points.

F) Determine the constant shear flow due to the torque acting at the shear center.

Comments Again the accuracy depends on a thin wall approximation.

G) Determine the normal skin stresses.

Comments Because the principal centroidal axis and the neutral axis are coincident the normal stress maybe simply calculated from the flexural formula.

ONLY BECAUSE LOADS ARE RESOLVED INTO COMPONENTS IN THE PRINCIPAL DIRECTIONS.

H) Determine the twist and deflection

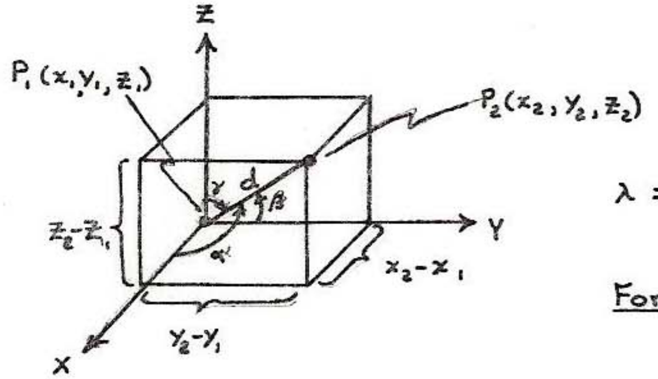
$$\frac{d^2\ \text{Displacement}}{d\ \text{Span}^2} = \frac{M}{EI} \qquad \theta \doteq \frac{1}{2(\text{area})^2 G} \sum \frac{qS}{t}$$

Comments The differentil equation could be solved by using the fourth order runge-kutta numerical method. The flexural rigity (EI) for the skin is equal to the flexural rigidity of the composite. Shear deformations are shown to be small with respect to deformations due to bending, Appendix C.

Unfortunately the runge-kutta and adams-multon methods of solving the differential equations were time consuming. Instead the solutions for deflections for point and distributed loads where used. Starting at the wing root the wing is divided into as many sections as desired for accuracy. Each section is treated as a freebody of constant crossection. The deflections for each section due to moment and shear loads where accumulated using superposition as the program progressed from the root to the tip. Twisting deflections were calculated for each section as a function of the shear flow distribution in the skin due to the torque load.

ESSEN TIALLY A FIN ELEM PROCE

DIRECTION COSINES FOR AERODYNAMIC CENTER LINE

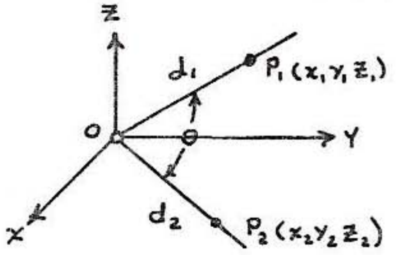


Direction Cosines Defined

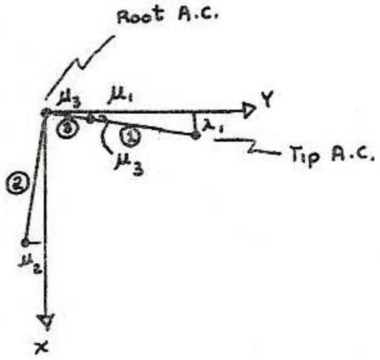
$$\lambda = \frac{x_2-x_1}{d} \qquad \mu = \frac{y_2-y_1}{d} \qquad \nu = \frac{z_2-z_1}{d}$$

For two lines

$$\text{Cos}\,\Theta = \lambda_1\lambda_2 + \mu_1\mu_2 + \nu_1\nu_2$$



Determination of Direction Cosines
For
Aerodynamic Center Line



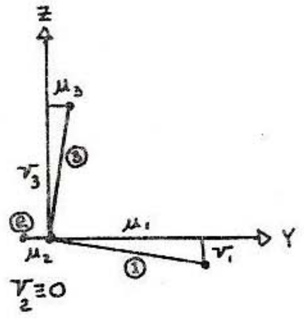
X, Y, Z => Reference axis

λ_1, μ_1, ν_1 calculated from Root and Tip A.C. locations

$$\left(\mu_1\mu_2 = -\lambda_1\lambda_2\right) \Rightarrow \frac{\mu_2}{\lambda_2} = -\frac{\lambda_1}{\mu_1}$$

$$\left(\lambda_2^2 + \mu_2^2 = 1\right) \Rightarrow \lambda_2 = \sqrt{\frac{1}{(\mu_2/\lambda_2)^2 + 1}}$$

$$\mu_2 = -\frac{\lambda_1}{\mu_1}\lambda_2 \qquad \nu_2 \equiv 0$$



$$\left(\mu_2\mu_3 = -\lambda_2\lambda_3\right) \Rightarrow \frac{\lambda_3}{\mu_3} = -\frac{\mu_2}{\lambda_2}$$

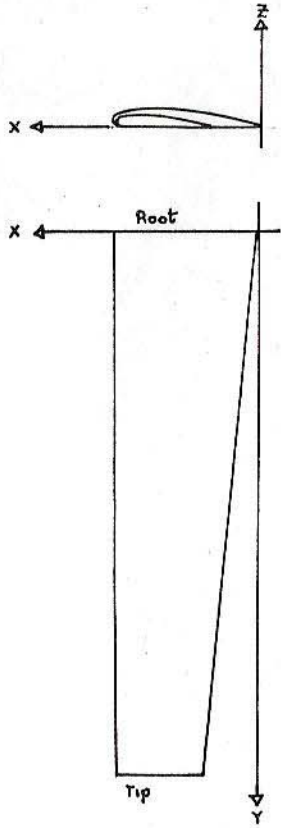
$$\left(\mu_1\mu_3 + \lambda_1\lambda_3 + \nu_1\nu_3 = 0\right) \Rightarrow \frac{\lambda_3}{\nu_3} = \frac{\nu_1 \frac{\lambda_3}{\mu_3}}{\lambda_1 \frac{\lambda_3}{\mu_3} + \mu_1}$$

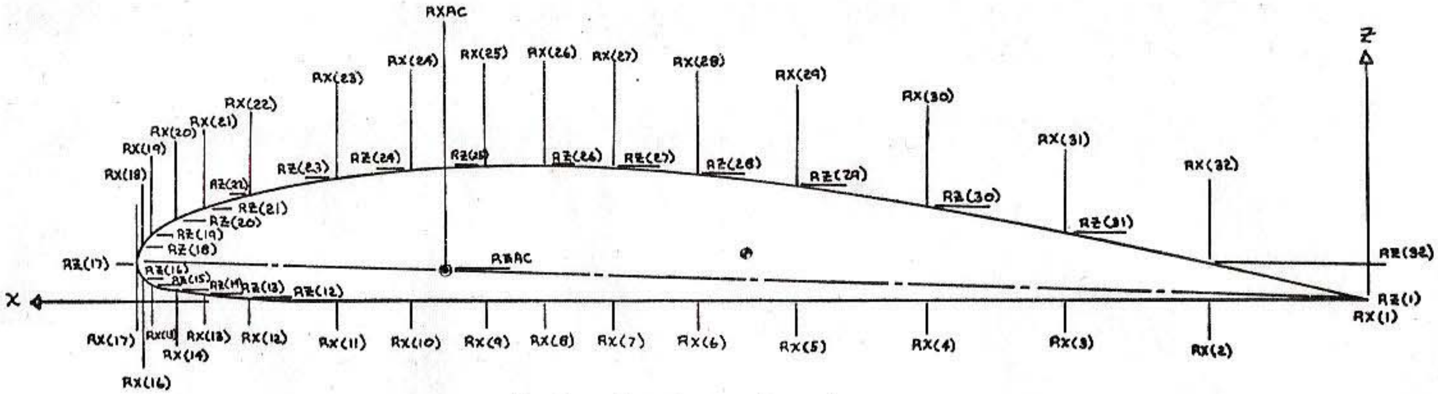
$$\left(\frac{\mu_3}{\nu_3} = -\frac{\nu_1}{\mu_1}\right)$$

$$\left(\mu_3^2 + \lambda_3^2 + \nu_3^2 = 1\right) \quad \nu_3 = \sqrt{\frac{1}{\left(\frac{\lambda_3}{\nu_3}\right)^2 + \left(\frac{\mu_3}{\nu_3}\right)^2 + 1}}$$

$$\left(\frac{\lambda_3}{\mu_3} = \frac{\lambda_1}{\mu_1}\right) \qquad \mu_3 = -\nu_3\,\nu_1/\mu_1 \,;\; \lambda_3 = \mu_3\,\lambda_1/\mu_1$$

AIRFOIL SHAPE DETERMINED AT ANY SPAN LOCATION



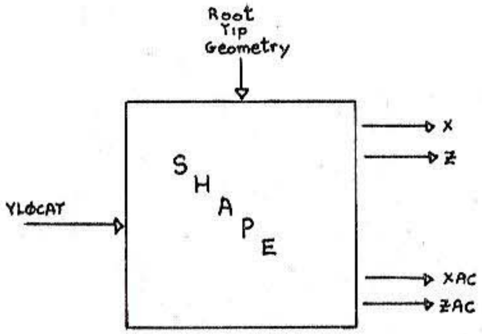


Equation For a Line in three dimensions

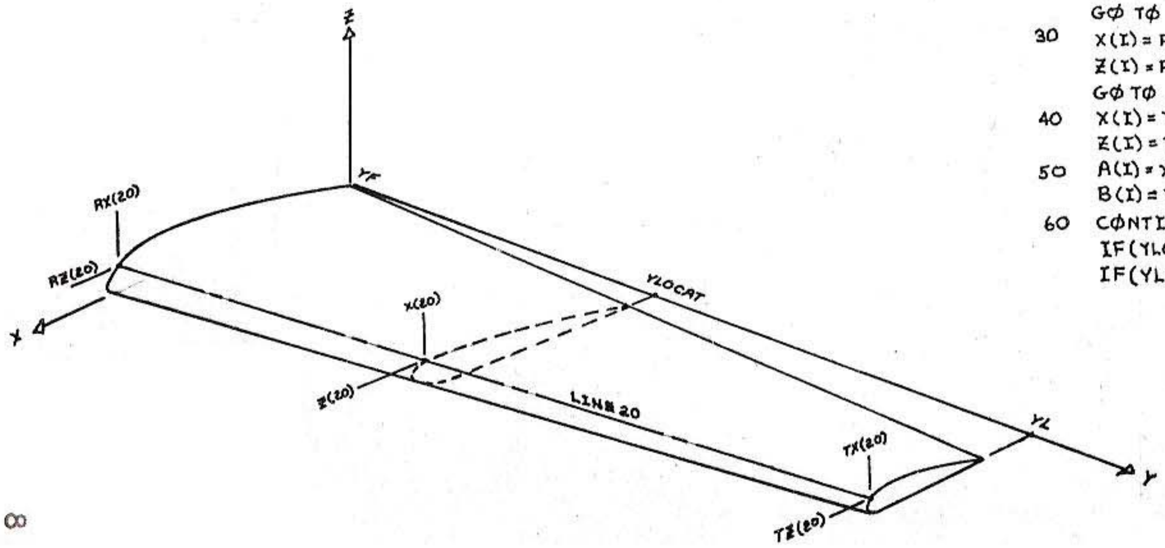
$$\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}$$

For line 20

$$\frac{X(20)-RX(20)}{TX(20)-RX(20)}=\frac{YLOCAT-YF}{YL-YF}=\frac{Z(20)-RZ(20)}{TZ(20)-RZ(20)}$$



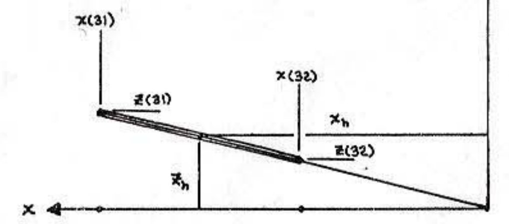
```
      SUBRØUTINE  SHAPE (RX,RZ,TX,TZ,RXAC,RZAC,TXAC,TZAC,YF,YL,YLØCAT,X,Z,XAC,ZAC,A,B)
      DIMENSIØN RX(32),RZ(32),TX(32),TZ(32),X(32),Z(32),A(32),B(32)
      DØ  60 I=1,32
      IF(YLØCAT.EQ.YF) GØ TØ 30
      IF(YLØCAT.EQ.YL) GØ TØ 40
      X(I) = ((TX(I)-RX(I))*(YLØCAT-YF)/(YL-YF)) + RX(I)
      Z(I) = ((TZ(I)-RZ(I))*(YLØCAT-YF)/(YL-YF)) + RZ(I)
      GØ TØ 50
30    X(I) = RX(I)
      Z(I) = RZ(I)
      GØ TØ 50
40    X(I) = TX(I)
      Z(I) = TZ(I)
50    A(I) = X(I)
      B(I) = Z(I)
60    CØNTINUE
      IF(YLØCAT.EQ.YF) GØ TØ 70
      IF(YLØCAT.EQ.YL) GØ TØ 80
                XAC = ((TXAC-RXAC)*(YLØCAT-YF)/(YL-YF)) + RXAC
                ZAC = ((TZAC-RZAC)*(YLØCAT-YF)/(YL-YF)) + RZAC
                GØTØ 90
          70    XAC = RXAC
                ZAC = RZAC
                GØTØ90
          80    XAC = TXAC
                ZAC = TZAC
          90    RETURN
                END
```



DETERMINE THE LOCATION OF THE PRINCIPAL CENTROIDAL AXIS

and

MOMENTS OF INERTIA



$\Delta x = |x(32) - x(31)| \qquad x_h = \Delta x/2$

$\Delta z = |z(32) - z(31)| \qquad z_h = \Delta z/2$

$d = \sqrt{\Delta x^2 + \Delta z^2}$

$\sin\phi = \frac{\Delta z}{d} \quad \cos\phi = \frac{\Delta x}{d}$

$I_{x_s} = \frac{dt^3}{12}$

$I_{z_s} = \frac{dt^3}{12} \qquad I_{xz_s} = 0$

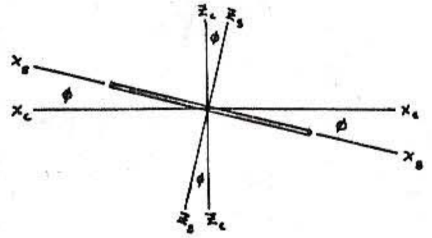
$A = dt$

$I_{x_c} = I_{x_s}(\cos\phi)^2 + I_{z_s}(\sin\phi)^2 - 2\cancelto{0}{I_{xz_s}}\sin\phi\cos\phi$

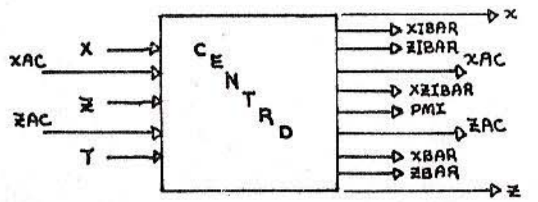
$I_{z_c} = I_{x_s}(\sin\phi)^2 + I_{z_s}(\cos\phi)^2 + 2\cancelto{0}{I_{xz_s}}\sin\phi\cos\phi$

$I_x = I_{x_c} + A z_h^2$

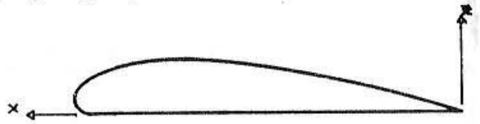
$I_z = I_{z_c} + A x_h^2$

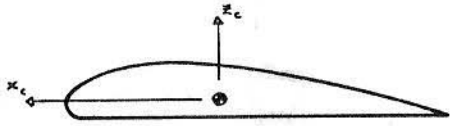


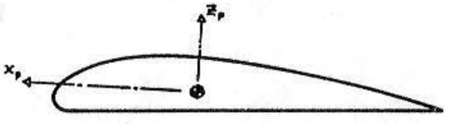
Portion	A	x_h	z_h	Ax	Az	Axz	I_{xc}	I_{z_c}	Ax^2	Az^2	I_x	I_z
1												
2												
⋮												
31												
32												
	ΣA			ΣAx	ΣAz	ΣAxz					ΣI_x	ΣI_z



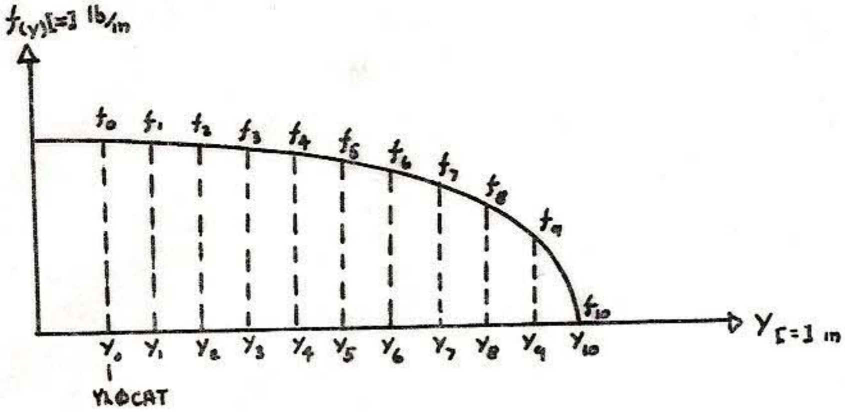
```
      SUBRØUTINE CENTRD(X,Z,T,XIBAR,ZIBAR,XZIBAR,XBAR,ZBAR,XAC,ZAC,TANG,PMI)
      DIMENSIØN X(32),Z(32),XP(32),ZP(32)
      TANG=0.0
      ANG=0.0
    1 AREAT=0
      AXT=0
      AZT=0
      AXZT=0
      XIT=0
      ZIT=0
      DØ10 I=1,32
      J=I+1
      IF(I.EQ.32) J=1
      DELTAX = X(J)-X(I)
      DELTAZ = Z(J)-Z(I)
      DIST = SQRT(DELTAX**2 + DELTAZ**2)
      XIS = DIST*(T**3)/12.0
      ZIS = T*(DIST**3)/12.0
      SINANG = DELTAZ/DIST
      CØSANG = DELTAX/DIST
      XIC = XIS*(CØSANG**2)+ZIS*(SINANG**2)
      ZIC = XIS*(SINANG**2)+ZIS*(CØSANG**2)
      AREA = DIST*T
      XBARS=(X(J)+X(I))/2.0
      ZBARS=(Z(J)+Z(I))/2.0
      AX = AREA*XBARS
      AZ = AREA*ZBARS
      AXZ = AREA*XBARS*ZBARS
      AXSQD = AREA*(XBARS**2)
      AZSQD = AREA*(ZBARS**2)
      XI = XIC+AZSQD
      ZI = ZIC+AXSQD
      AREAT = AREA+AREAT
      AXT= AX+AXT
      AZT= AZ+AZT
      AXZT = AXZ+AXZT
      XIT = XI+XIT
      ZIT= ZI+ZIT
   10 CØNTINUE
      CXBAR = AXT/AREAT
      CZBAR= AZT/AREAT
      IF(ANG.EQ.0.0) XBAR=CXBAR
      IF(ANG.EQ.0.0) ZBAR=CZBAR
      XIBAR=XIT-AREAT*(CZBAR**2)
      ZIBAR=ZIT-AREAT*(CXBAR**2)
      XZIBAR = AXZT-AREAT*CXBAR*CZBAR
      PMI = XIBAR+ZIBAR
      Y = 2.*XZIBAR
      W = ZIBAR-XIBAR
      IF(Y.LT.0.001)RETURN
      ANG = ATAN2(Y,W)/2.
      TANG = TANG+ANG
      DØ20 I=1,32
      XP(I)= X(I)*CØS(ANG)+Z(I)*SIN(ANG)
   20 ZP(I)=-X(I)*SIN(ANG)+Z(I)*CØS(ANG)
      XACP = XAC*CØS(ANG)+ZAC*SIN(ANG)
      ZACP =-XAC*SIN(ANG)+ZAC*CØS(ANG)
      DØ 30 I=1,32
      X(I) = XP(I)
   30 Z(I) = ZP(I)
      XAC= XACP
      ZAC= ZACP
      GØTØ1
      END
```







NUMERICAL DETERMINATION OF THE BENDING MOMENT, SHEAR FORCE, AND TORQUE



$$\text{Force} = \int_a^b f_{(y)}\,dy = \sum_{i=0}^{n} \frac{H}{2}(f_1 + f_{i+1}) = \frac{H}{2}\left[f_0 + 2f_1 + 2f_2 + \cdots\cdots + 2f_{n-1} + f_n\right] \ [=]\ \text{lb}$$

$$\text{Moment} = \int_a^b f_{(y)}\,y\,dy = \sum_{i=0}^{n} \frac{f_i + f_{i+1}}{2}\,H\,\frac{y_i + y_{i+1}}{2} \quad [=]\ \text{lb.in.}$$

$$\begin{aligned} = \frac{H}{4}\Big[\, & f_0 y_0 + f_0 y_1 + f_1 y_0 \\ & + 2f_1 y_1 + f_1 y_2 + f_2 y_1 \\ & + 2f_2 y_2 + f_2 y_3 + f_3 y_2 \\ & + \cdots\cdots \\ & \cdots + 2f_{n-1} y_{n-1} + f_{n-1} y_n + f_n y_{n-1} \\ & + f_n y_n \Big] \end{aligned}$$

SHEAR FLOW IN MULTI-CELLED THIN-WALLED SECTTONS

It is assumed that all material in the wing is effective in resisting shear loads. Because of nonsymmetric geometry the easiest method used (from the stand point of programing) to calculate shear flow distribution is the method of successive approximations, as outlined in Bruhn.[1] This method is used for calculating shear flow distributions due to torsion loads and shear loads. Representative calculations are shown in the example problem.

V. EXAMPLE PROBLEM

To check for accuracy and to facilitate debugging an example problem was programed before inputting the wing geometry. The example problem was constructed symmetric so that analytic hand calculations were simple. These analytical calculations would then be compared with computer calculations. Unfortunately the computer program has not been debugged and the comparison is pending. The detailed calculations and results are shown in Appendix A. The computer program listing is in Appendix C.

VI. STATIC STRESS ANALYSIS OF THE LIBELLE WING

Once the computer program is debugged then the wing geometry, Figure 6, and material properties, Appendix B, can be inputted into the computer program.

VII. STRESS-STRAIN RELATIONSHIPS FOR THIN ORTHOTROPIC LAMINATED PLATES

The skin on the Libelle wing is a ±45° symmetric fiber/resin laminate 0.030 inches thick. Such a thin skin can be idealized as having three planes of symmetry (orthotropic). If the fiber/resin geometry is ignored then the skin can be considered quasi-homogeneous. If it is assumed that a state of plane stress exists then the anisotropic stress-strains relations are reduced to a simplified form. Where [Q] is called the stiffness matrix and [S] is called the compliance matrix.[2]

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{21} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} , \quad \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{21} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

where

$$Q_{11} = E_{11} / (1 - \nu_{12} \nu_{21})$$

$$Q_{22} = E_{22} / (1 - \nu_{21} \nu_{12})$$

$$Q_{12} = \nu_{12} E_{11} / (1 - \nu_{12} \nu_{21})$$

$$Q_{66} = G_{12}$$

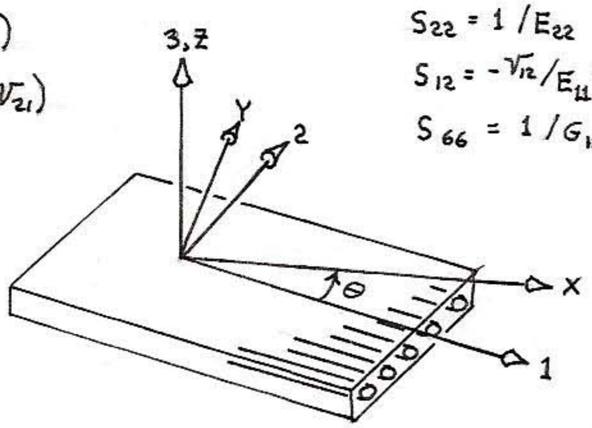
where

$$S_{11} = 1 / E_{11}$$

$$S_{22} = 1 / E_{22}$$

$$S_{12} = -\nu_{12} / E_{11} = -\nu_{21} / E_{22}$$

$$S_{66} = 1 / G_{12}$$



If one is interested in stress-strain relationships along an axis x,y rotated θ from the laminate axis 1,2, then the transformed stiffness and compliance matrix, labelled $[\tilde{Q}]$ and $[\tilde{S}]$ respectively, are shown below as functions of θ and the original [Q] and [S] matrix.

$$n = \sin\Theta \qquad m = \cos\Theta$$

$$\bar{Q}_{11} = m^4 Q_{11} + 2m^2n^2 Q_{12} + 4m^3n\, Q_{16} + n^4 Q_{22} + 4mn^3 Q_{26} + 4m^2n^2 Q_{66}$$

$$\bar{Q}_{12} = m^2n^2 Q_{11} + (m^4+n^4)\, Q_{12} + (2mn^3 - 2m^3n)\, Q_{16} + m^2n^2 Q_{22} + (2m^3n - 2mn^3)\, Q_{26} - 4m^2n^2 Q_{66}$$

$$\bar{Q}_{16} = -m^3n\, Q_{11} + (m^3n - 2mn^3)\, Q_{12} + (m^4 - 3m^2n^2)\, Q_{16} + mn^3 Q_{22} + (3m^2n^2 - n^4)\, Q_{26} + (2m^3n - 2mn^3)\, Q_{66}$$

$$\bar{Q}_{22} = n^4 Q_{11} + 2m^2n^2 Q_{12} - 4mn^3 Q_{16} + m^4 Q_{22} - 4m^3n\, Q_{26} + 4m^2n^2 Q_{66}$$

$$\bar{Q}_{26} = -mn^3 Q_{11} + (mn^3 - m^3n)\, Q_{12} + (3m^2n^2 - n^4)\, Q_{16} + m^3n\, Q_{22} + (m^4 - 3m^2n^2)\, Q_{26} + (2mn^3 - m^3n)\, Q_{66}$$

$$\bar{Q}_{66} = m^2n^2 Q_{11} - 2m^2n^2 Q_{12} + (2mn^3 - 2m^3n)\, Q_{16} + m^2n^2 Q_{22} + (2m^3n - 2mn^3)\, Q_{26} + (m^2 - n^2)\, Q_{66}$$

$$\bar{S}_{11} = m^4 S_{11} + 2m^2n^2 S_{12} + 2m^3n\, S_{16} + n^4 S_{22} + 2mn^3 S_{26} + m^2n^2 S_{66}$$

$$\bar{S}_{12} = m^2n^2 S_{11} + (m^4+n^4)\, S_{12} + (mn^3 - m^3n)\, S_{16} + m^2n^2 S_{22} + (m^3n - mn^3)\, S_{26} - m^2n^2 S_{66}$$

$$\bar{S}_{16} = -2m^3n\, S_{11} + (2m^3n - 2mn^3)\, S_{12} + (m^4 - 3m^2n^2)\, S_{16} + 2mn^3 S_{22} + (3m^2n^2 - n^4)\, S_{26} + (m^3n - mn^3)\, S_{66}$$

$$\bar{S}_{22} = n^4 S_{11} + 2m^2n^2 S_{12} - 2mn^3 S_{16} + m^4 S_{22} - 2m^3n\, S_{26} + m^2n^2 S_{66}$$

$$\bar{S}_{26} = -2mn^3 S_{11} + (2mn^3 - 2m^3n)\, S_{12} + (3m^2n^2 - n^4)\, S_{16} + 2m^3n\, S_{22} + (m^4 - 3m^2n^2)\, S_{26} + (mn^3 - m^3n)\, S_{66}$$

$$\bar{S}_{66} = 4m^2n^2 S_{11} - 8m^2n^2 S_{12} + (4mn^3 - 4m^3n)\, S_{16} + 4m^2n^2 S_{22} + (4m^3n - 4mn^3)\, S_{26} + (m^2 - n^2)^2 S_{66}$$

One can plot the variation of E_{xx} as a function of Θ simply by taking the reciprocal of $\bar{S}_{11}$. A plott of $\bar{Q}_{11}$ and $1/\bar{S}_{11}$ for a boron-epoxy laminate is shown in Figure 5. These results favorably compare with a similiar example problem in the "Advanced Composite Design Guide".[3] This method was used to calculate a modulus of elasticity and shear modulus oriented 45° from the laminate fiber axis for the skin on the Libelle wing.

$$E_{xx45^0} = 1.42 \times 10^6 \text{ psi} \qquad G_{xy45^0} = 6.18 \times 10^5 \text{ psi}$$

The material properties E_{11}, E_{22}, v_{12}, G_{12} which were used to calculate E_{xx} at 45° from the laminate axis were obtianed by tensile testing wing skin sections on the instron tensile testing machine and recording longitudinal and transverse strains, Appendix B. Well defined experimental testing techniques were used to determine these material properties.[4]

These material properties were also used to calculate stresses along the laminate axis for the eight rossette strain gages located around the wing as shown in Figure 3,4. The stresses located at the x,y axis rotated θ from the 1,2 axis were calculated using the 2nd order stress transformation relation, Table III.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & (\cos^2\theta-\sin^2\theta) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

All of the above was programmed to reduce data from the 24 strain gages, shown in figures 3,4, and to determine a modulus of elasticity and shear modulus along the wing Y axis as shown in Figure 6. The computer program listing is in Appendix D.

VIII. SHEAR STRESS DISTRIBUTION

Because of limited time only shear flow distribution was expermentally verified. Deflections could have been verified but the author felt this would have been easily demonstrated. More questionable variables due to computer program idealization were choosen to be investigated. The questionable computer program idealization was the elimination of the balsa core. This idealization, although reasonable for bending and torsional resistance, is questionable with respect to shear flow distribution. The shear center location is calculated from the shear flow distribution. The shear center is a very important variable with respect to a static and dynamic structual analysis. To statically analyze the wing due to bending and torsion the shear center must be located so that bending and torsion could be decoupled and the effects of each considered seperately. Sailplane wings, because of their long slender geometry, are subject suspect to flutter. In a flutter analysis if the equations of motion for bending and twisting are to be decoupled then the shear center location must be determined.

The shear stress distribution around a particular airfoil crossection was experimentally determined by locating straingages at a section 18 inches from the root airfoil as shown in Figures 3,4. Since the largest strains occur at the root the strain gages were placed as close to the root airfoil as possible for best reading accuracy range but sufficiently removed from root end effects.

The stress-strain relationships for orthotropic laminate thin skins in plane stress were used to ? obtain normal and shear stresses at the plane 18 inches from the root. The strain gages were oriented along the fiber axis of the composite. composite's fiber axis. The material properties along the laminate axis could be determined. The stresses at the rotated plane of interest is calculated by using the 2nd order tensor transformation matrix (Mohr's circle).

Also of interest was the variation of Youngs modulus and shear modulus along an axis rotated θ degrees from the laminate axis, Figure 5. Normal and shear modului for the x,y wing axis at the section 18 inches from the root are needed for calculations in the computer program. All of the above was programmed. The results are shown in Table III. The material properties are listed in Table II.

IX. RESULTS AND DISCUSION

Upon completion of the computer program the calculated shear stress distribution can be compared with the experimental shear stress distribution, Table III.

Experimental youngs modulus 45° from the laminate axis, Appendix B, does not favorably compare with the analytic calculated youngs modulus, $E_{xx}^{45°} = 1.41 \times 10^6$, Figure 5. Unfortunately the modulus increased by the amount that it should have decreased at 45° rotation from the laminated axis 1,2. To check for this discrepency an example problem with known results were compared with those results calculated by the computer program. There was no notable difference.

It should be noted that the computer static stress analysis assumes that the wing skin is thin and isotropic while the libelle wing is orthotropic and definitely not thin at the spar cap, see Figure 1. These approximations are expected to cause no large differences between analytic and experimental results but until the computer program is debugged little discussion on this can be made.

X. CONCLUSIONS

A method for experimentally measuring normal and shear stress distributions for an orthotropic thin skin was formulated and programmed.

A computer program was written to statically stress analyze sailplane wings whose construction is similiar to the open class Libelle wing . Unfortunately this program has not been debugged. Until this program is working no meaningful conclusions can be made on the static stress analysis.

XI. REFERENCES

1. Bruhn E.F., Analysis And Design Of Flight Vehicle Structures, Tri-state Offset Company, 1965.

2. Ashton J.E., Halpin, Petit P.H., Primer on Composite Materials Analysis, Technomic, 1969.

3. Rockwell International's "Advanced Composite Design Guide", Los Angeles Aircraft Division, Rockwell International, Los Angeles, 1973.

4. Rosen B.W., A simple procedure for Experimental Determination of the Longitudinal Shear Modulus of Unidirectional Composites, Journal of Composite Materials, Vol. 6(October 1972).p. 552.

XII. FIGURES AND TABLES

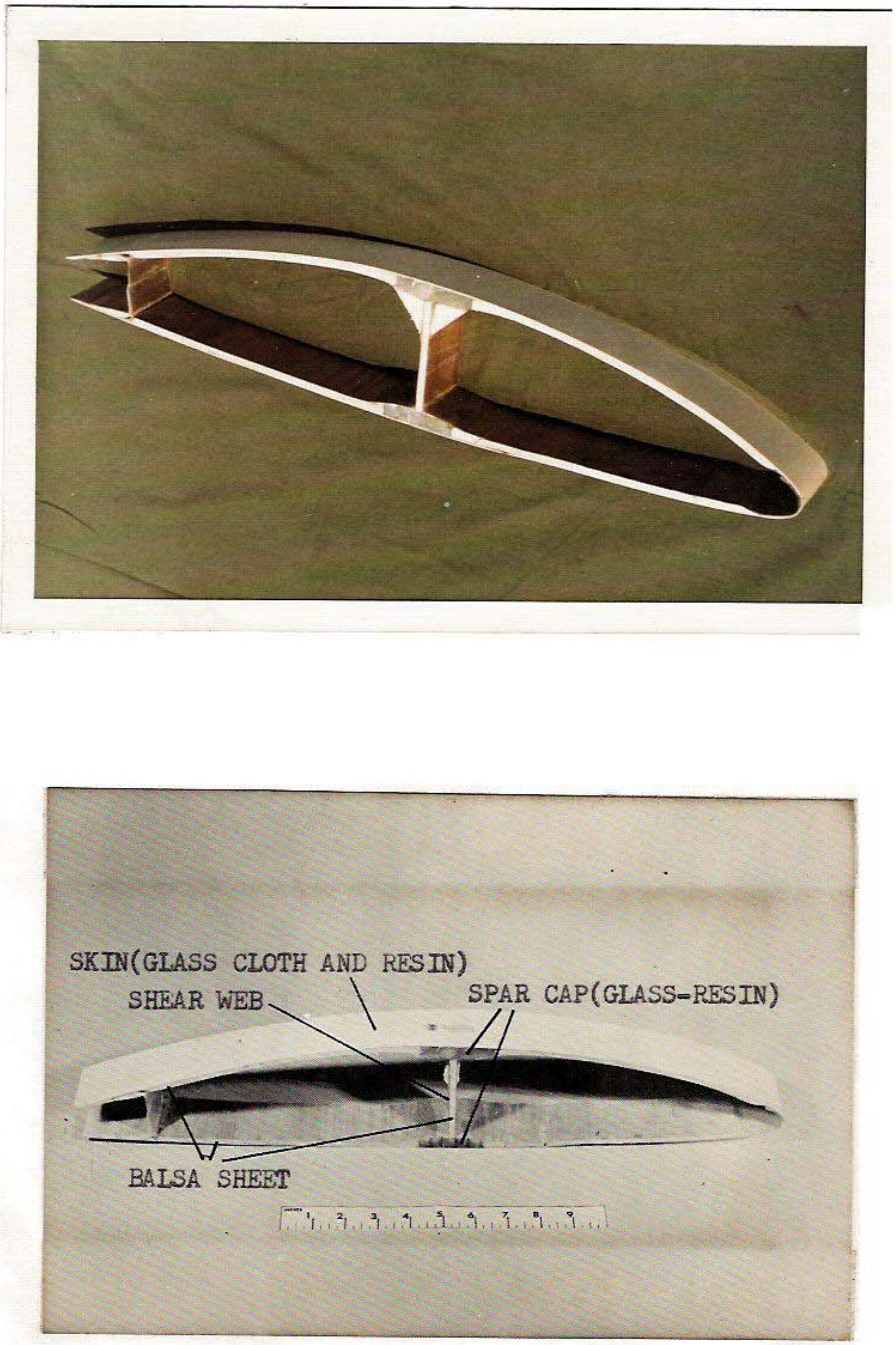
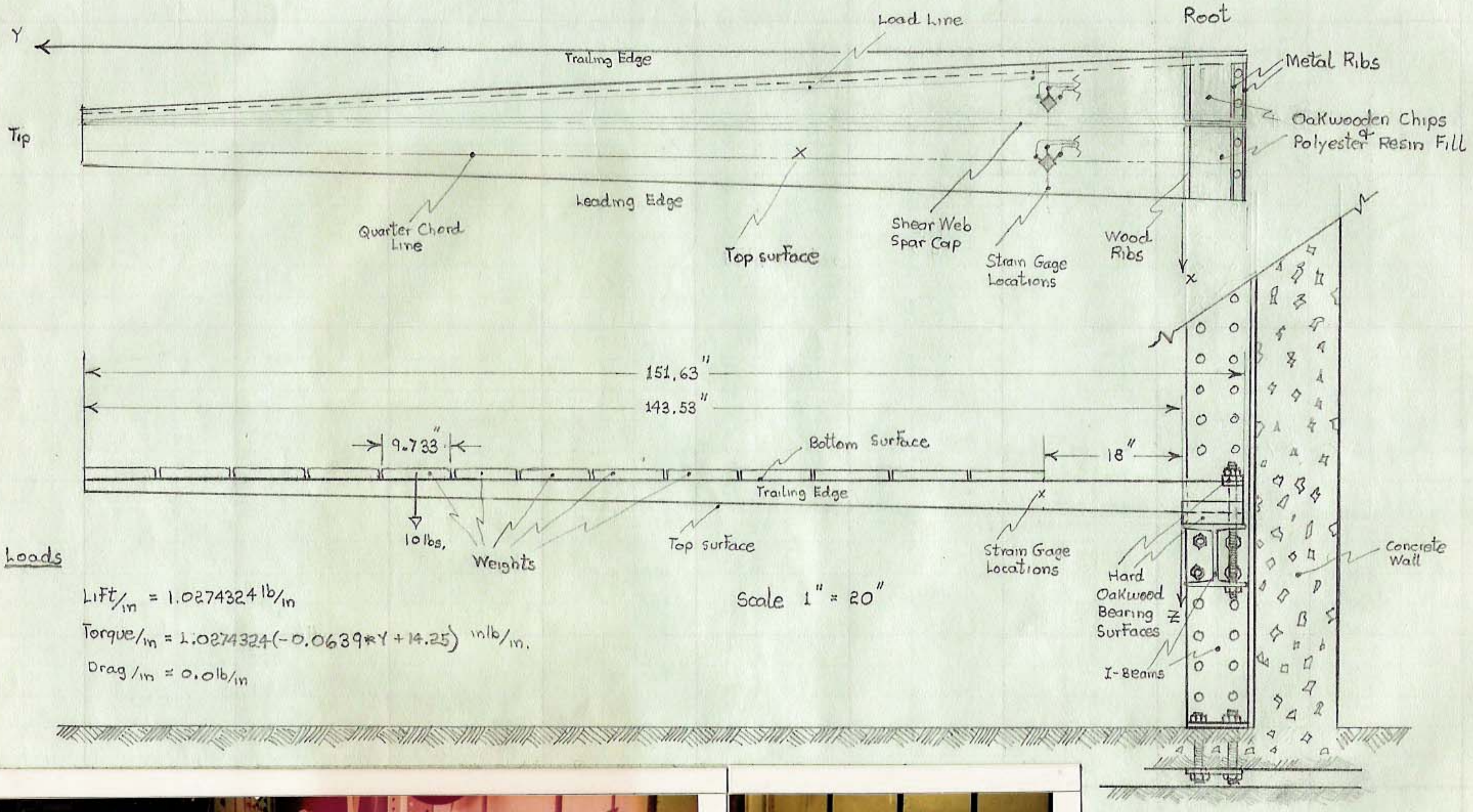


Figure 1. Open Class Libelle Sailplane Wing Construction



Loads

$Lift/_{in} = 1.0274324\ ^{lb}/_{in}$

$Torque/_{in} = 1.0274324(-0.0639*Y + 14.25)\ ^{in lb}/_{in.}$

$Drag/_{in} = 0.0\ ^{lb}/_{in}$

Figure 2

Wing test Setup

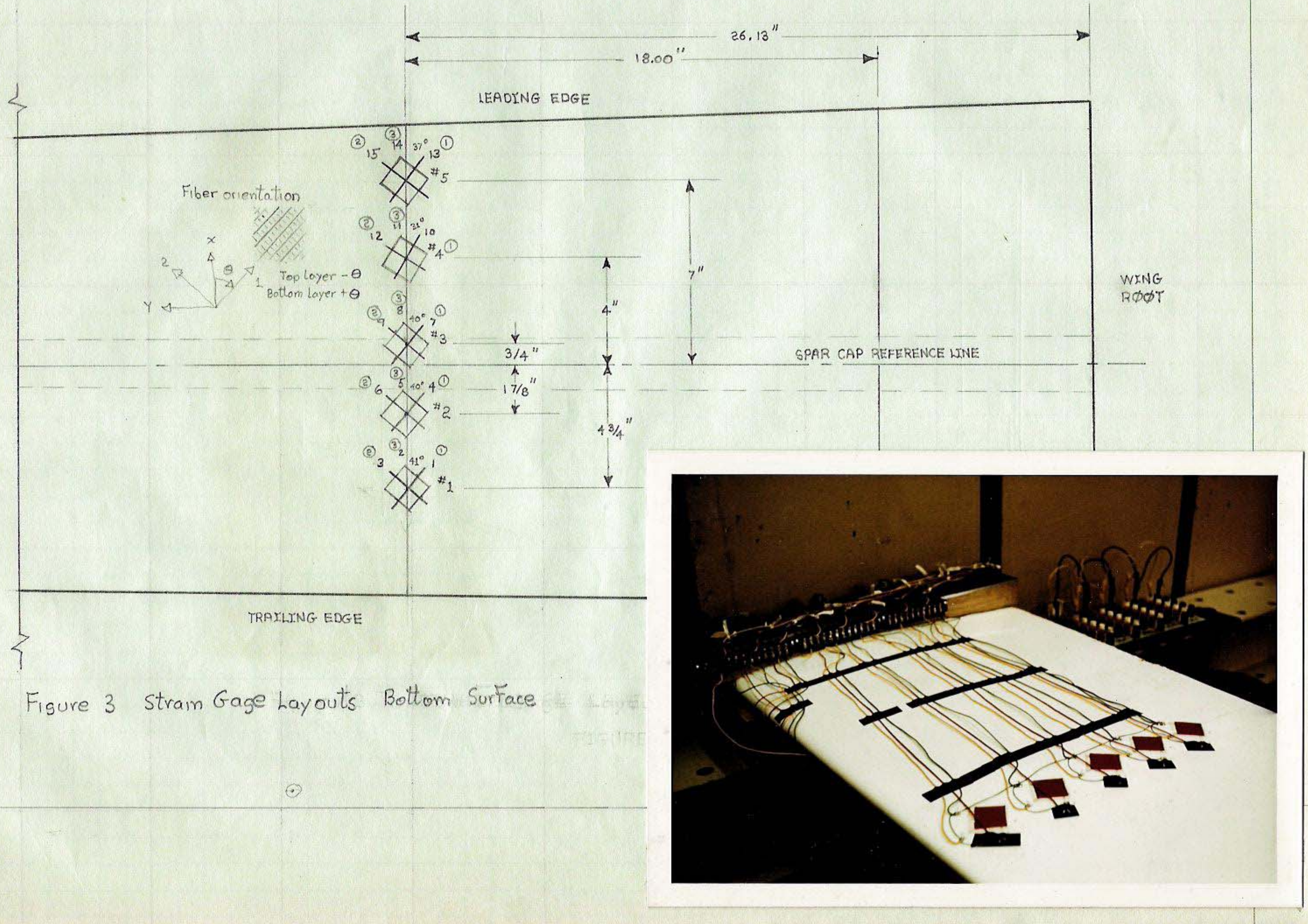


Figure 3 Strain Gage Layouts Bottom Surface

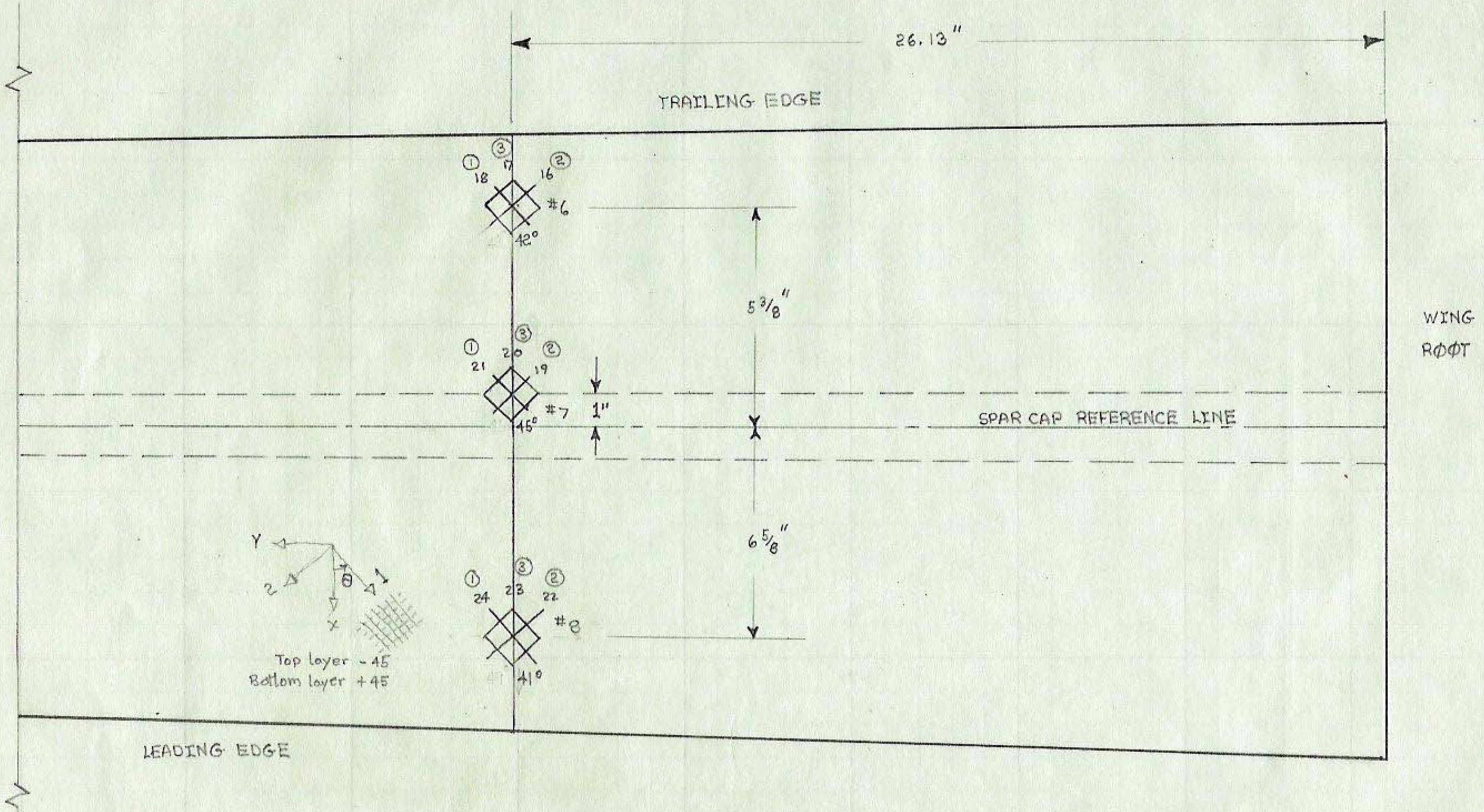


Figure 4. Strain Gage Layout Top surface.

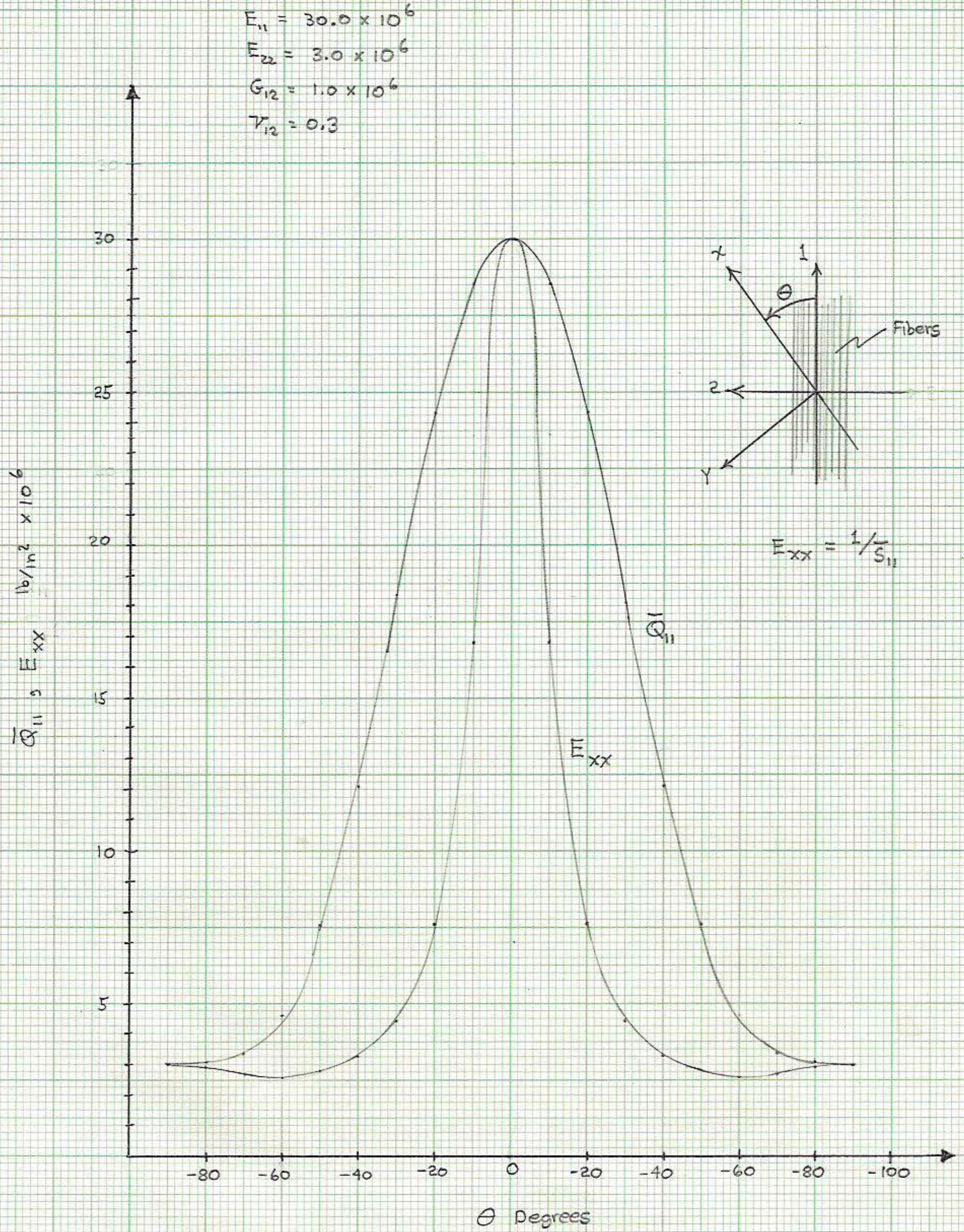


Figure 5. Variation of $\bar{Q}_{11}$ and E_{xx} with Filament Orientation

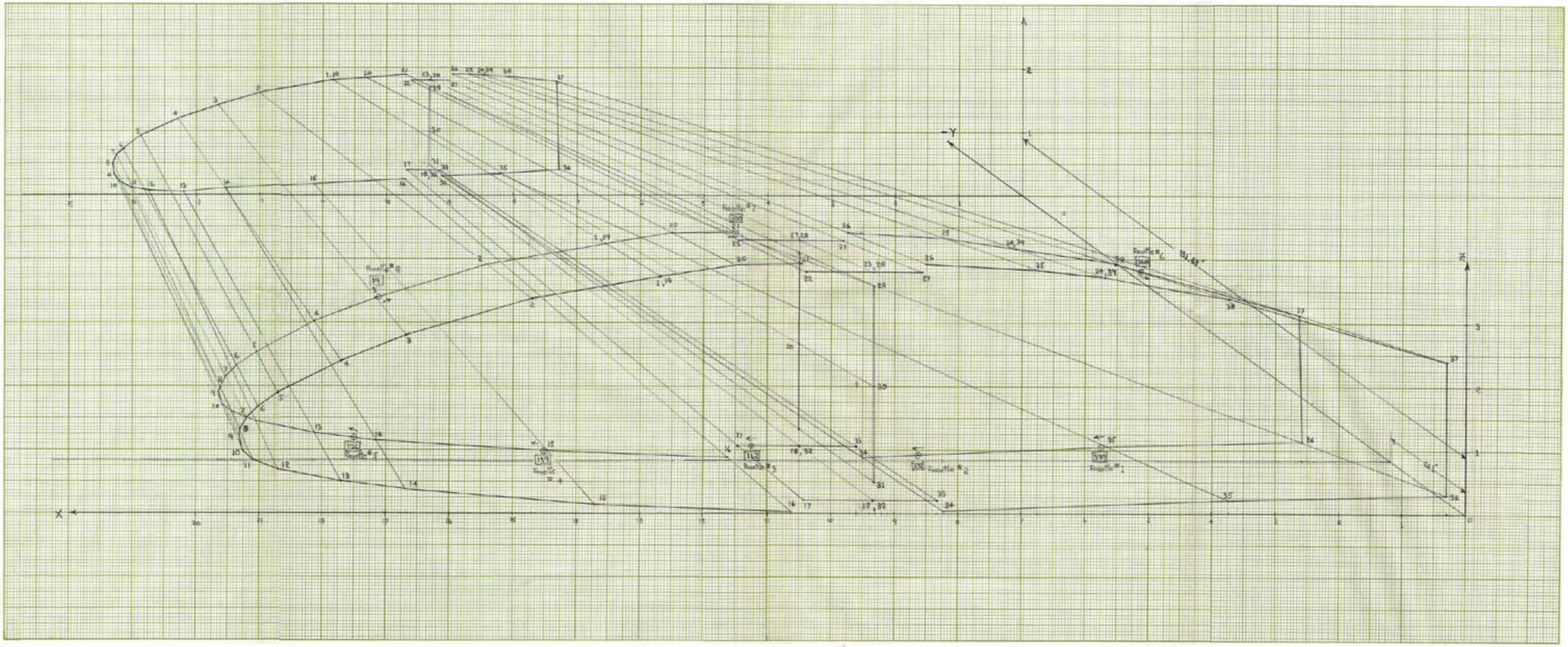
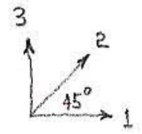


Figure 6. Libelle Wing Geometry

Table I

Strains At The 18 Inches Airfoil Crossection

Rossette No.	Strain Gage No.	Strain $^{\mu}$in/in	Angle Rotation in Degrees From the 1 Fiber Axis to the airfoil crossectional plane
1	1	200	41
	2	-17	
	3	-263	
2	4	217	40
	5	52	
	6	-220	
3	7	165	40
	8	160	
	9	-194	
4	10	39	31
	11	232	
	12	222	
5	13	68	37
	14	131	
	15	-281	
6	16	-148	~~48~~ 42
	17	-158	
	18	357	
7	19	52	45
	20	-209	
	21	249	
8	22	-55	~~49~~ 41
	23	-139	
	24	236	



Equipment - Strain gage type, paper base, 60R, G.F. 1.97, lot No. 34
- Recording instruments Budd Indicator and Switch&Balance Unit

Table II

Material Properties For the Skin and Spar Cap of the Open Class Libelle Sailplane Wing

		E_{11} lb/in^2 X 10^6	E_{22} lb/in^2 X 10^6	G_{12} lb/in^2 X 10^6	ν_{12}	ν_{21}
Skin		$\approx$ 1.33	1.33	0.722	0.0583	0.0583
Spar Cap	Tension	5.74	0.336	0.46	0.342	
	Compression	$\approx$ 3.40	–	–	–	–

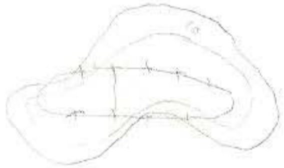


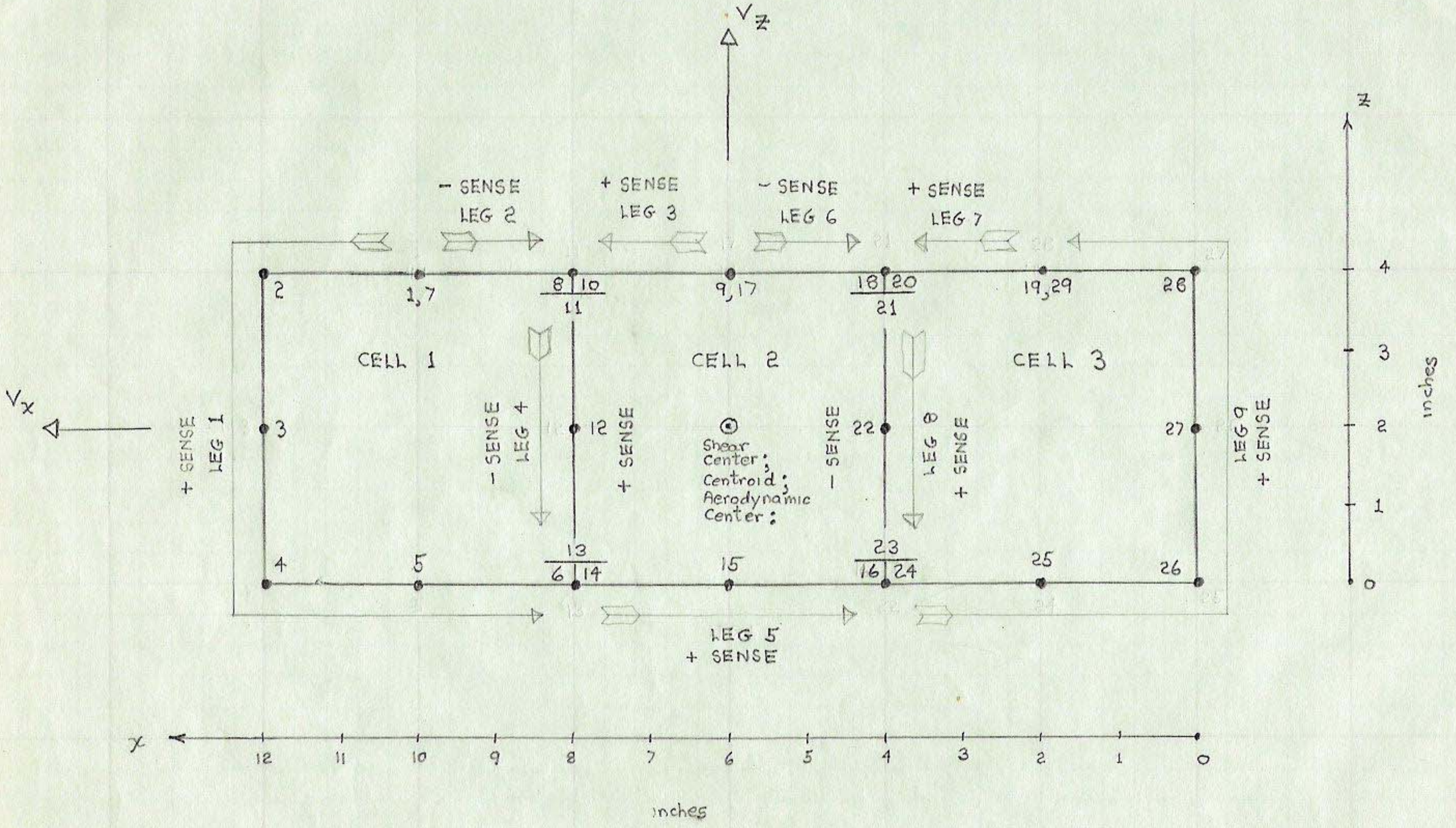
Table III

Experimental Normal & Shear Stress Distribution For an Open Class Libelle Wing At the 18 inch Airfoil Crossection

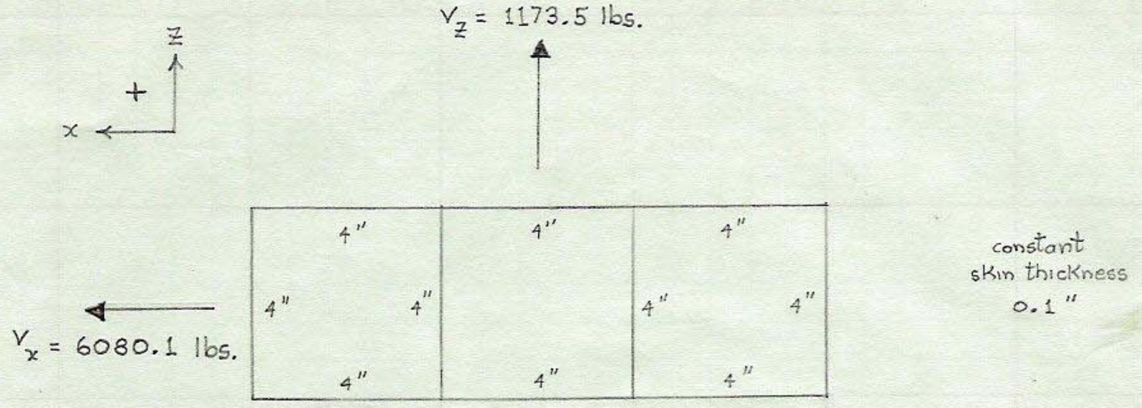
Rossette No		Normal Stress lb/in^2			Shear Stress lb/in^2					
1	339	1732	616	1599	23.5	-204	22.6	8	21.6	2.30
2	332	1845	673	1701	26.0	-189	18.9	8	18.2	1.59
3	162	1678	732	1549	23.8	-92.4	23.7	1	23.3	1.95
4	139	1709	700	1622	6.78	-137	16.3	2	16.1	1.75
5	126	1431	677	1370	12.9	-114	12.8	2	12.5	1.27
6	368	- 1433	-945	- 1461	- 25.0	-83.1	25.3	6	24.2	- 0.436
7	200	- 2231	-582	- 2078	- 18.3	-162	18.9	.6	18.4	- 20.0
8	19	- 1139	-604	- 998.9	- 12.5	-118	11.4	4	10.9	- 1.26
			an	isotropic		ex an		an is an		

Appendix A

Example Problem



V_Z
- SENSE
LEG 2
+ SENSE
LEG 3
- SENSE
LEG 6
+ SENSE
LEG 7
2
1,7
8 | 10
11
9,17
18 | 20
21
19,29
28
CELL 1
CELL 2
CELL 3
V_X
+ SENSE
LEG 1
3
- SENSE
LEG 4
12
+ SENSE
Shear Center;
Centroid;
Aerodynamic Center;
- SENSE
22
LEG 8
+ SENSE
27
LEG 9
+ SENSE
4
5
13
6 | 14
15
23
16 | 24
25
26
LEG 5
+ SENSE
z
4
3
2
1
0
inches
x
12
11
10
9
8
7
6
5
4
3
2
1
0
inches



Moments of inertia

$$I_x \text{ (For top + Bottom sKins)} = \bar{I} + Ad^2 = 2\left\{\frac{12(0.1)^3}{12} + 12(0.1)(2)^2\right\} = 9.602 \text{ in}^4$$

$$I_x \text{ (webs)} = 4\left\{\frac{0.1(4)^3}{12}\right\} = 2.133 \text{ in}^4$$

$$\underline{\underline{I_x = 11.735 \text{ in}^4}}$$

$$I_z \text{ (Sides)} = 2\left\{\frac{0.1(12)^3}{12}\right\} = \frac{172.8}{6} = 28.8 \text{ in}^4$$

$$I_z \text{ (Top + Bottom)} = 2\left\{\frac{4(0.1)^3}{12} + 4(0.1)(6)^2\right\} = 28.80067 \text{ in}^4$$

$$I_z \text{ (Top + Bottom interior)} = 2\left\{\frac{4(0.1)^3}{12} + 4(0.1)(2)^2\right\} = 3.20067 \text{ in}^4$$

$$\underline{\underline{I_z = 60.801 \text{ in}^4}}$$

Shear Open Cell Calculations
Due to Shear Load V_z

$$q_{i+1} = q_i - \frac{V_z}{I_x} \sum_{n=i}^{i+1} z_{\substack{cen\\ i,i+1}} A_{i \to i+1}$$

Leg $_1$

$q_1 = 0$

$q_{10} = q_1 - 100\{2[2(0.1)]\} = -40$

$q_{20} = q_{10} - 100\{1[2(0.1)]\} = -60$

$q_{30} = q_{20} - 100\{-1[2(0.1)]\} = -40$

$q_{40} = q_{30} - 100\{-2[2(0.1)]\} = 0$

$q_{50} = q_{40} - 100\{-2[2(0.1)]\} = +40$

Stop

Leg $_2$

$q_{60} = 0$

$q_{70} = q_{60} - 100\{2[2(0.1)]\} = -40$

Stop

Leg $_3$

$q_{80} = 0$

$q_{90} = q_{80} - 100\{2[2(0.1)]\} = -40$

Leg $_4$

$q_{100} = q_{70} + q_{90} = -80$

$q_{110} = q_{100} - 100\{1(0.2)\}$

$q_{120} = q_{110} - 100\{-1(0.2)\}$

Stop

Leg $_5$

$q_{130} = q_{50} + q_{120} = -40$

$q_{140} = q_{130} - 100\{-2(0.2)\} = 0$

$q_{150} = q_{140} - 100\{-2(0.2)\} = +40$

Stop

Leg $_6$

$q_{160} = 0$

$q_{170} = q_{160} - 100\{2(0.2)\} = -40$

Stop

Leg $_7$

$q_{180} = 0$

$q_{190} = q_{180} - 100\{2(0.2)\} = -40$

Stop

Leg 8

$q_{200} = q_{170} + q_{190} = -80$

$q_{210} = q_{200} - 100\{1(0.2)\} = -100$

$q_{220} = q_{210} - 100\{-1(0.2)\} = -80$

stop

Leg 9

$q_{230} = q_{150} + q_{220} = -40$

$q_{240} = q_{230} - 100\{-2(0.2)\} = 0$

$q_{250} = q_{240} - 100\{-2(0.2)\} = +40$

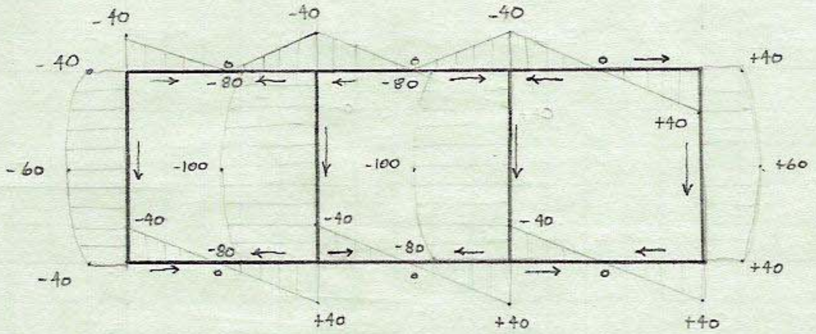
$q_{260} = q_{250} - 100\{-1(0.2)\} = +60$

$q_{270} = q_{260} - 100\{1(0.2)\} = +40$

$q_{280} = q_{270} - 100\{2(0.2)\} = 0$

stop

V_Z Open Cell Shear Flow



Closed Cell Shear Flow Calculations

$\Sigma q_i L/t_{cell}$	1600	0	-1600
$\Sigma L/t$ cell	160	160	160
$\Sigma L/t$ web	40	40	
C.O.F.	0.25	0.25 0.25	0.25
$q_c = -\Sigma q_i L/t / \Sigma L/t_{cell}$	-10	0	+10
C.O.	0	-2.5 +2.5	0
C.O.	0	0 0	0
$q_c + \Sigma$C.O.	-10	0	+10

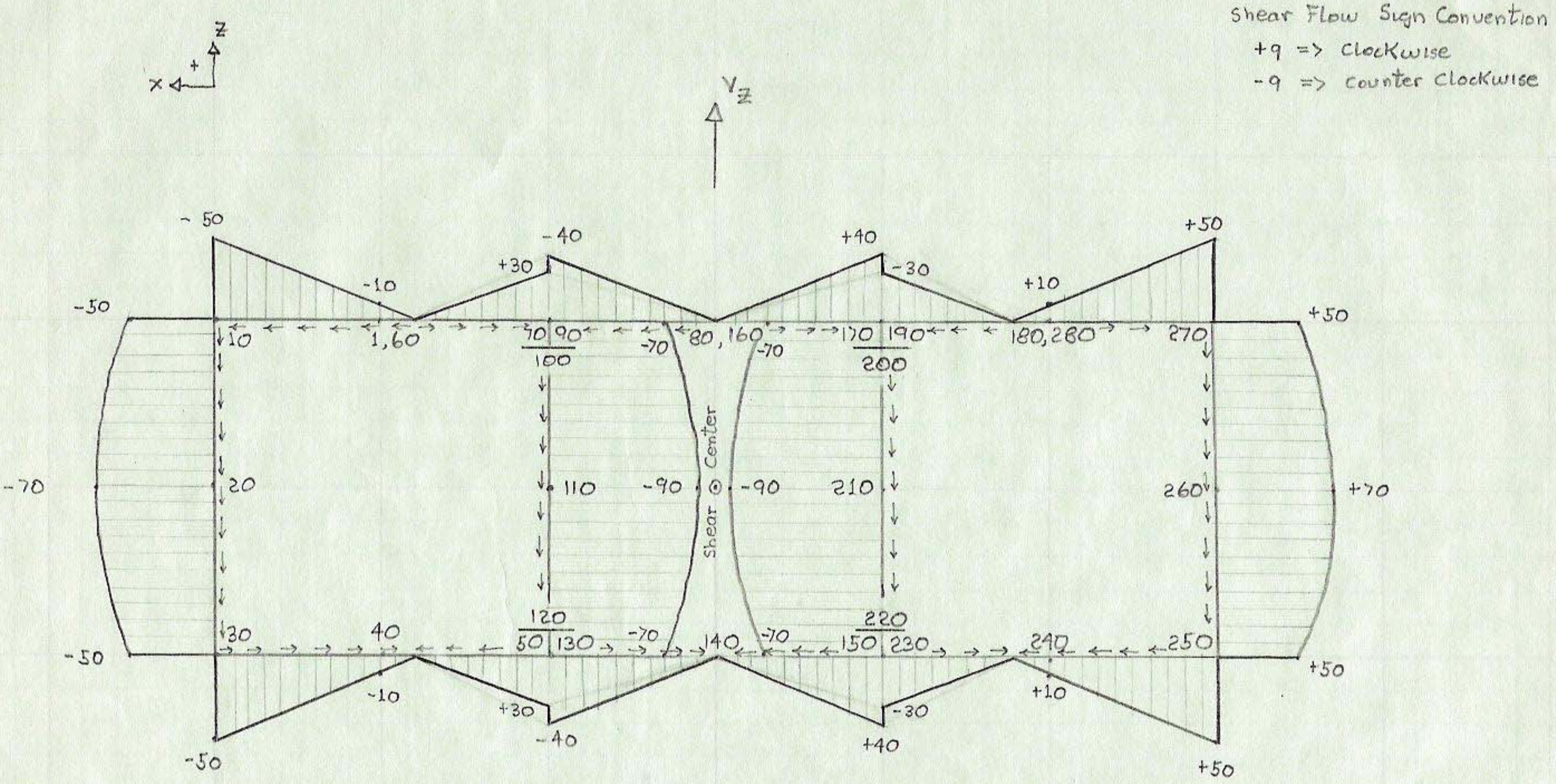
$\Sigma q_i L/t_{cell}$

where $q_i = q_i$ (SENSE SIGN)

Cell 1 $\Sigma q_i L/t_1 = \left[-40\frac{4}{0.1} + (-20)\frac{(4)^{2/3}}{0.1}\right] + \left[80\frac{4}{0.1} + 20\frac{(4)^{2/3}}{0.1}\right] = 1600$

Cell 2 $\Sigma q_i L/t_2 = 0$

Cell 3 $\Sigma q_i L/t_3 = \left[-80\frac{4}{0.1} + (-20)\frac{(4)^{2/3}}{0.1}\right] + \left[40\frac{4}{0.1} + 20\frac{(4)^{2/3}}{0.1}\right] = -1600$


Shear Flow Sign Convention
+q => Clockwise
-q => counter Clockwise
Z
x
+
Y_Z
- 50
-10
+30
-40
+40
-30
+10
+50
-50
+50
10
1,60
70
90
100
-70
80,160
-70
170
190
200
180,280
270
-70
20
110
-90
Shear Center
-90
210
260
+70
120
220
30
40
50
130
-70
140
-70
150
230
240
250
-50
+50
-10
+30
-40
-30
+40
+10
-50
+50

Shear Open Cell Calculations
Due to Shear Load V_x

$$q_{i+1} = q_i - \frac{V_x}{I_z}\sum_{n=i}^{i+1} x_{cen} A_{i\to i+1}$$

Leg$_1$

$q_1 = 0$

$q_{10} = q_1 - 100\{5(0.2)\} = -100$

$q_{20} = q_{10} - 100\{6(0.2)\} = -220$

$q_{30} = q_{20} - 100\{6(0.2)\} = -340$

$q_{40} = q_{30} - 100\{5(0.2)\} = -440$

$q_{50} = q_{40} - 100\{3(0.2)\} = -500$

Stop

Leg$_2$

$q_{60} = 0$

$q_{70} = q_{60} - 100\{3(0.2)\} = -60$

Stop

Leg$_3$

$q_{80} = 0$

$q_{90} = q_{80} - 100\{1(0.2)\} = -20$

Stop

Leg$_4$

$q_{100} = q_{70} + q_{90} = -80$

$q_{110} = q_{100} - 100\{2(0.2)\} = -120$

$q_{120} = q_{110} - 100\{2(0.2)\} = -160$

Stop

Leg$_5$

$q_{130} = q_{50} + q_{120} = -660$

$q_{140} = q_{130} - 100\{1(0.2)\} = -680$

$q_{150} = q_{140} - 100\{-1(0.2)\} = -660$

Leg$_6$

$q_{160} = 0$

$q_{170} = q_{160} - 100\{-1(0.2)\} = +20$

Stop

Leg 7

$q_{180} = 0$

$q_{190} = q_{180} - 100\{-3(0.2)\} = +60$

Stop

Leg$_8$

$q_{200} = q_{170} + q_{190} = +80$

$q_{210} = q_{200} - 100\{-2(0.2)\} = +120$

$q_{220} = q_{210} - 100\{-2(0.2)\} = +160$

Stop

Leg$_9$

$q_{230} = q_{150} + q_{220} = -500$

$q_{240} = q_{230} - 100\{-3(0.2)\} = -440$

$q_{250} = q_{240} - 100\{-5(0.2)\} = -340$

$q_{260} = q_{250} - 100\{-6(0.2)\} = -220$

$q_{270} = q_{260} - 100\{-6(0.2)\} = -100$

$q_{280} = q_{270} - 100\{-5(0.2)\} = 0$

Closed Cell Calculations

$\sum q_i {}^{L}/_{t_{cell}}$ where $q_i = q_i$ (SENSE SIGN)

Cell$_1$ $\sum q_i {}^{L}/_{t_1} = \frac{-100(2)2}{3(0.1)} + \frac{-220(4)}{0.1} + \frac{-340(4)}{0.1} + \frac{-160(4)2}{3(0.1)} + \frac{60(2)2}{3(0.1)} + \frac{120(4)}{0.1} = -22400.00$

Cell$_2$ $\sum q_i {}^{L}/_{t_2} = \frac{-20(2)2}{3(0.1)} + \frac{-120(4)}{0.1} + \frac{-500(4)}{0.1} + \frac{-180(4)2}{3(0.1)} + \frac{-20(2)2}{3(0.1)} + \frac{-120(4)}{0.1} = -34933.33$

Cell$_3$ $\sum q_i {}^{L}/_{t_3} = \frac{60(2)2}{3(0.1)} + \frac{120(4)}{0.1} + \frac{-340(4)}{0.1} + \frac{-160(4)2}{3(0.1)} + \frac{-220(4)}{0.1} + \frac{-100(2)2}{3(0.1)} = -22400.00$

266
3,800

+800.00
+4800.00
+5600.00

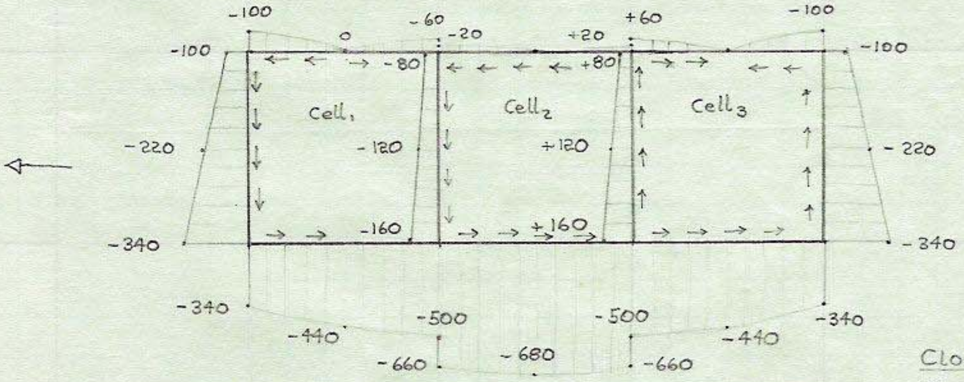
-1333.33
-8800.00
-13600.00
-4266.67
-28000.00
+5600.00
22400.00

$160\overline{)22400.00}$ 140.00

-266.66
-4800.00
20000.00
-4800.00
-266.66
-4800.00
34933.33

$160\overline{)34933.33}$ 218.58
320
293
160
1333
1240
933
800
1333

Shear Flow Open Cell



$\Sigma q_i {}^{L}/_{t}$ cell	-21600.00	-36800.00		-21600.00
$\Sigma {}^{L}/_{t}$ cell	160	160		160
$\Sigma {}^{L}/_{t}$ web	40		40	
C.O.F.	0.25	0.25	0.25	0.25
$q_c = -\Sigma q_i {}^{L}/_{t} / \Sigma {}^{L}/_{t_{cell}}$	+135.00	+230.00		+135.00
C.O.	57.5	33.75	33.75	57.5
C.O.	16.875	14.375	14.375	16.875
C.O.	7.1875	4.21875	4.21875	7.1875
C.O.	2.1093	1.7968	1.7968	2.1093
C.O.	0.8984	0.52733	.52733	0.8984
C.O.	0.26366	0.2246	.2246	0.26366

Closed Cell
Shear Flow
Calculations

C.O.	0.1123	0.065916	0.065916	0.1123
C.O.	0.032958	0.028075	0.0287075	0.032958
C.O.	0.014038	0.0082396	0.0082396	0.014038
$q_c + \Sigma$ C.O.	220.00	340.00		220.00

Displacements For Uniformly Distributed Loads

$$\text{Deflection}_{max} = \frac{WL^4}{8EI} = \frac{81.0706\,(100)^4}{(8)2.5\times10^6(40.5353)} = \frac{2\times10^8}{2\times10^7} = 10.0$$

$E = 2.5 \times 10^6$ psi

$$\text{Slope}_{max} = \frac{WL^3}{6EI} = \frac{81.0706\,(100)^3}{(6)2.5\times10^6(40.5353)} = \frac{2\times10^6}{15\times10^6} = 0.1333 \text{ radians}$$

Weight Calculations

Circunference length = 40 in.

Core thickness = 0.1 in.

Skin thickness = 0.1 in.

Total Volume = 800 in.^3

Skin density = 0.025 lb/ in.^3

Core density = 0.001 lb/ in.^3

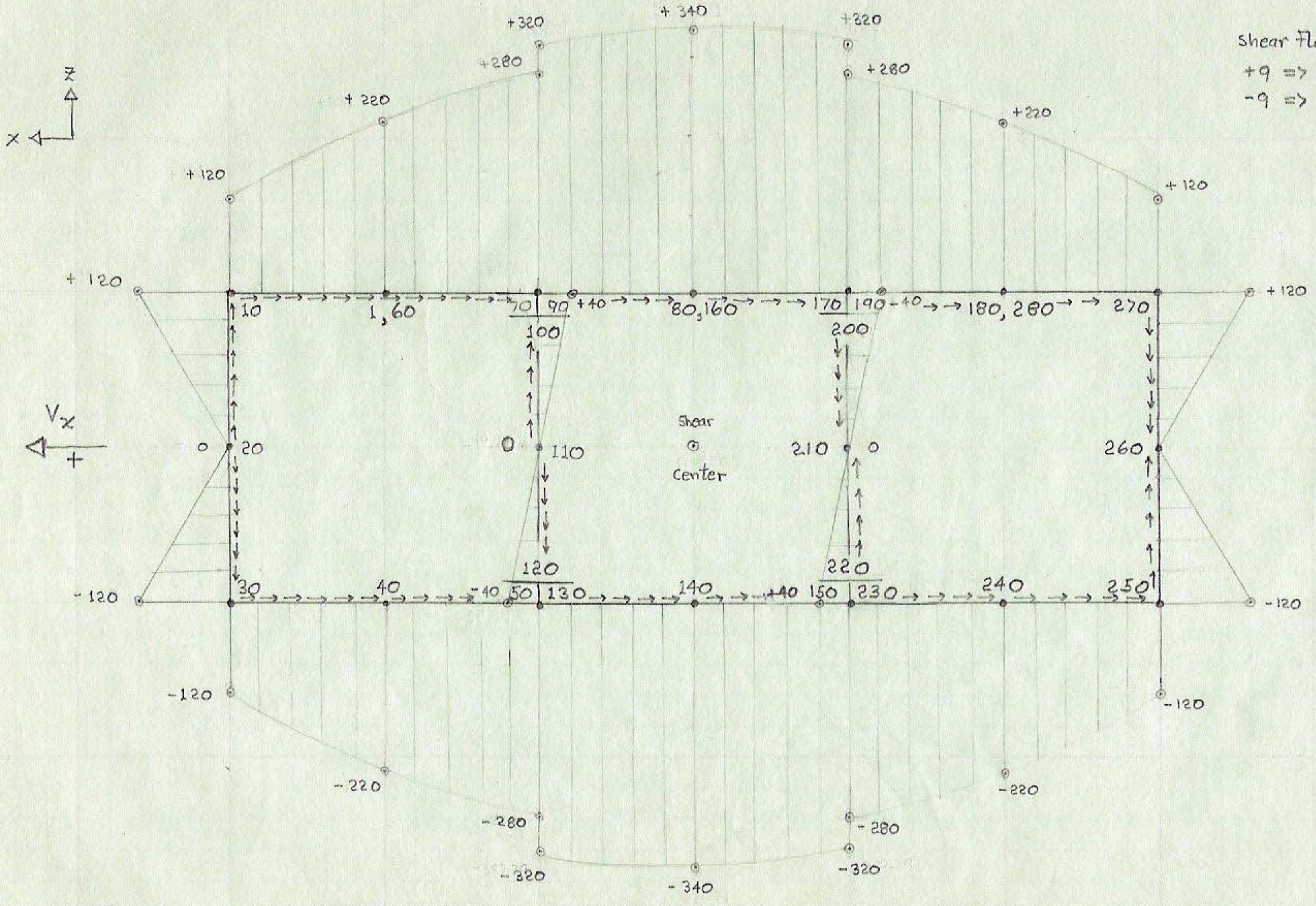
Beam Length = 100 in.

Core Volume = 400 in.^3

Skin Volume = 400 in.^3

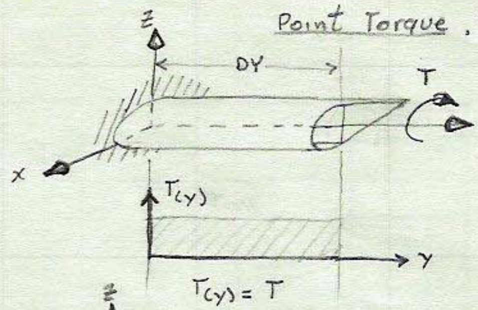
Skin Weight = 10 lb.

Core Weight = 0.4 lb.

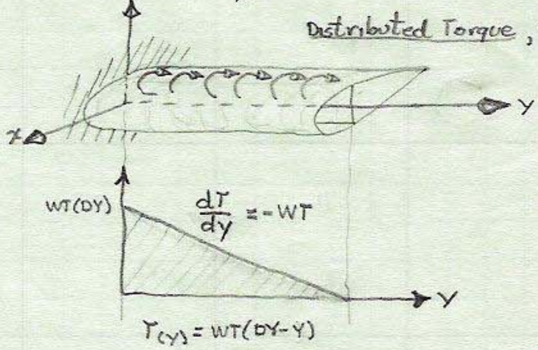


Shear Flow sign Convention
+q => Clockwise
-q => Counter Clockwise

Point Torque, T



Distributed Torque, WT



Point Torque, $T_{(y)} = \text{constant} = T$

$$U = \int_0^{DY} \oint \frac{q}{2Gt}\, ds\, dy$$

$$q = T/2A$$

$$T_{(y)} = \text{const.}$$

$$U = \int_0^{DY} \frac{T^2}{8A^2} \oint \frac{ds}{Gt}\, dy$$

$$U = \frac{T^2}{8A^2} \oint \frac{ds}{Gt} \int_0^{DY} dy$$

$$\theta_P = \frac{\partial U}{\partial T} = \frac{T}{4A^2G} \oint \frac{ds}{t} \int_0^{DY} dy$$

$$\frac{d\theta_P}{dy} = \frac{T}{4A^2G} \oint \frac{ds}{t} \quad ① \qquad \text{OR} \qquad \frac{d\theta_P}{dy} = \frac{1}{2AG} \oint \frac{q\,ds}{t} \quad ②$$

$$\frac{d\theta_P}{dy} = B\ T \quad ③ \qquad \text{where } B = \text{constant} \qquad \theta_P = \left[\frac{1}{2AG} \oint \frac{q\,ds}{t}\right] DY \quad ④$$

Successive approximation Iteration yields $\Rightarrow T, q_1, q_2, \ldots$

Calculate $\frac{d\theta_P}{dy}$ From $q_1, q_2, \ldots$ using ②

$$B = \frac{d\theta_P/DY}{T} = \frac{\theta_P/DY}{T} = \frac{\theta_P}{T(DY)}$$

Distributed Torque $\quad T_{(y)} = WT(DY - Y)$

$$\frac{d\theta_D}{dy} = B\ T_{(y)} = B(WT)(DY - Y)$$

$$\theta_D = \int_0^{DY} B(WT)(DY - Y)\, dy$$

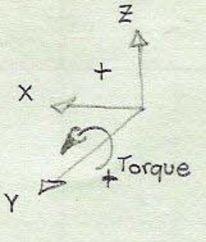
$$= B(WT)\left[(DY)Y - Y^2/2\right]\Big|_0^{DY}$$

$$= B(WT)\left[(DY)^2 - (DY)^2/2\right]$$

$$\theta_D = B(WT)\frac{DY^2}{2}$$

$$\theta_D = \frac{\theta_P}{T(DY)}(WT)\frac{DY^{\cancel{2}}}{2}$$

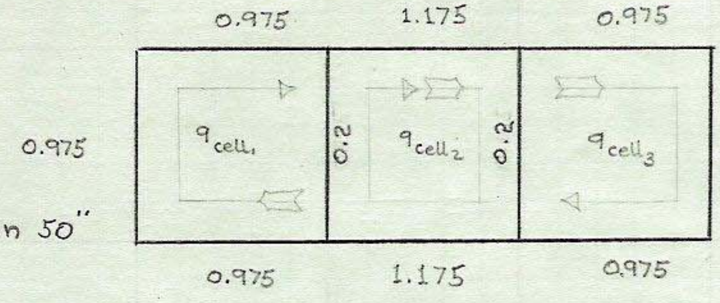
$$\theta_D = \frac{\theta_P(WT)(DY)}{2T}$$



$\overset{+}{\curvearrowleft}$ Torque applied = 100 inlb

$G\theta \equiv 1 \qquad q_{cell_1} = \frac{2A_{cell_1}}{\Sigma_{cell_1} L/t} = 0.2 \; {}^{lb}/_{in} = q_{cell_2} = q_{cell_3}$

$G = 3.42 \times 10^5$ psi



Crossection 50"

Shear Flow
Sign Convention
+q => Clockwise
−q => Counter Clockwise

WT = 2 in-lb/in

Length Total = 100 in. = L

2A	32	32		32
$\Sigma L/t$ cell	160	160		160
$\Sigma L/t$ web	40		40	
C.O.F	0.25	0.25	0.25	0.25
$q_c = 2A/\Sigma L/t$ cell	0.2	0.2		0.2
C.O.	0.05	0.05	0.05	0.05
C.O.	0.025	0.0125	0.0125	0.025
C.O.	0.00625	0.00625	0.00625	0.00625
C.O.	0.003125	0.0015625	0.0015625	0.003125
$q = q_c + \Sigma$ C.O.	0.284375	0.340625		0.284375
2Aq	9.1	10.9		9.1
T_{total} $G\theta \equiv 1$		29.1		
$q_{corrected}$	0.975	1.175		0.975

$\frac{T}{29.1} = \frac{100}{29.1} = \frac{q_{cell_1}}{0.2844} = \frac{q_{cell_2}}{0.341} = \frac{q_{cell_3}}{0.2844}$

At Crossection 50" Torque = 100 inlb

$$\theta_p = \left[\frac{1}{2AG} \oint \frac{q\,ds}{t}\right] DY \qquad \theta_D = \theta_p \frac{WT(DY)}{2T}$$

$$\theta_{T_{0''}} = 0 \quad \text{(Fixed end)}$$

$$\theta_p = \left[\frac{0.975(12) + (-0.2)4}{2(16)3.42\times10^{-5}\,0.1}\right] 50 \qquad \theta_D = 0.5\times10^{-3}\,\frac{2(50)}{2(100)}$$

$$\theta_p = 0.5\times10^{-3} \text{ radians} \qquad \theta_D = 0.25\times10^{-3} \text{ radians} \qquad \theta_{T_{50''}} = 0.75\times10^{-3} \text{ radians}$$

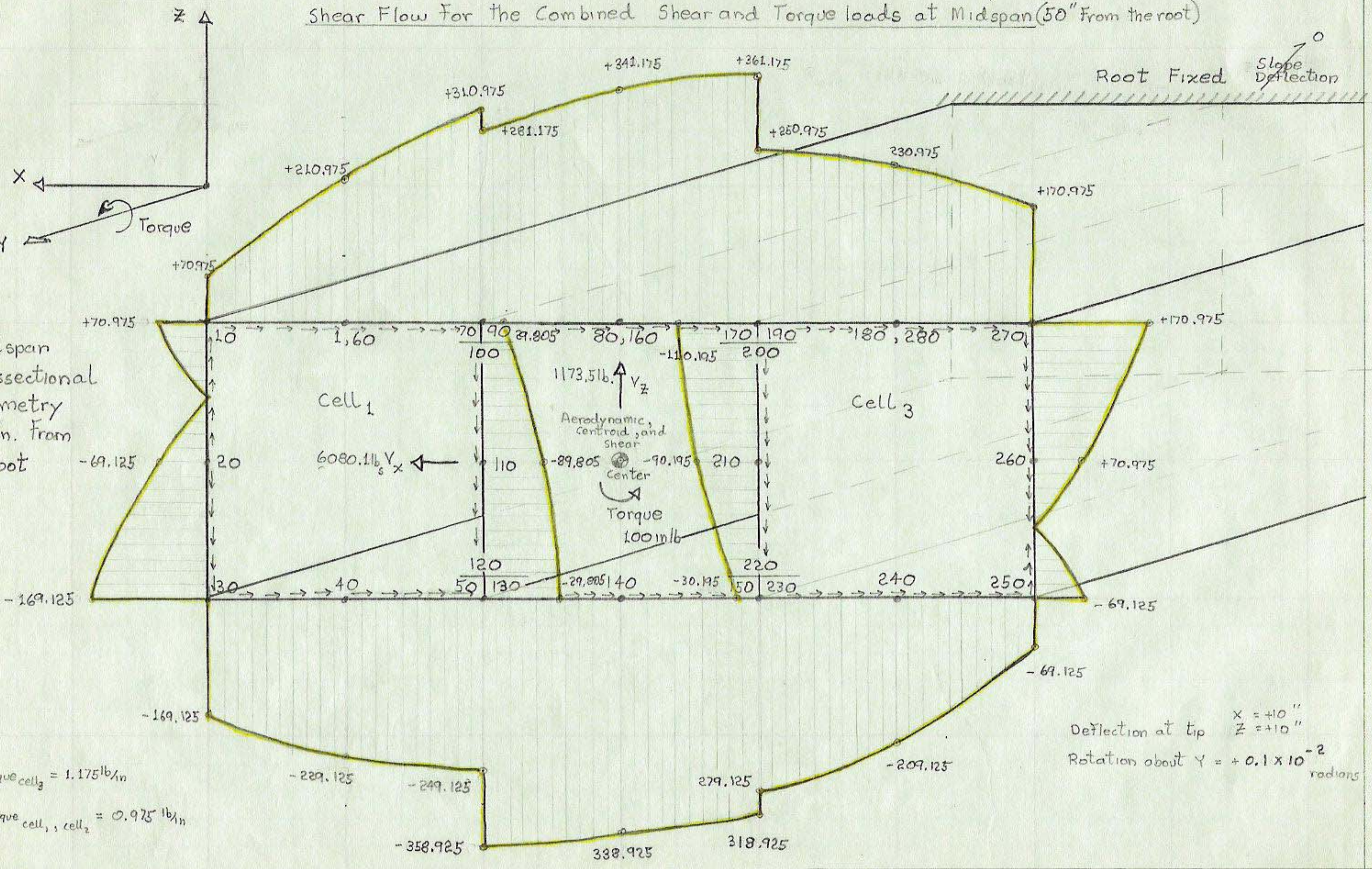
At Crossection 100" Torque = 0

$\theta_p = 0$ $\qquad \theta_D$ is the same since the crossectional geometry and distributed torque is the same over DY from 50" to 100"

$$\theta_D = 0.25\times10^{-3} \text{ radians}$$

$$\theta_{T_{100''}} = 0.1\times10^{-2} \text{ radians}$$

Shear Flow for the Combined Shear and Torque loads at Midspan (50" From the root)



Midspan
Crossectional
Geometry
50 in. From
Root

Deflection at tip $x = +10''$ $z = +10''$

Rotation about $Y = +0.1 \times 10^{-2}$ radians

$Q_{Torque_{cell_3}} = 1.175 \text{ lb/in}$

$Q_{Torque_{cell_1, cell_2}} = 0.975 \text{ lb/in}$

Appendix B

Experimental Determination of The Material Properties For The Libelle Wing

SKIN MØDULUS

INITIAL STRAIN μin/in

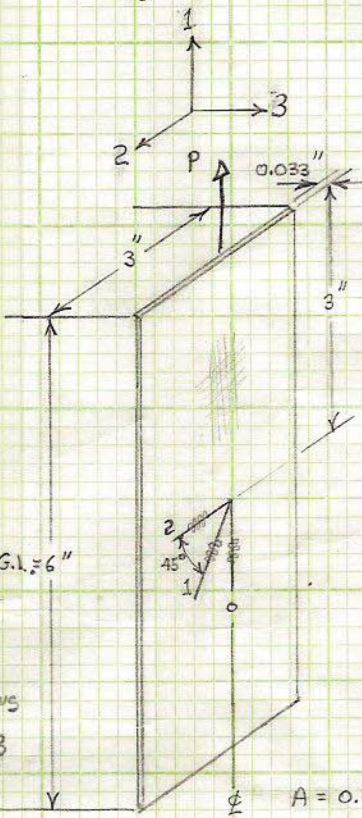
00 -997 -003
01 -992 -008
02 -995 -005

Load	STRAINS μin/in	corrected
42→38	00 +327	+330
	01 +184	+192
	02 -982	-023
81→79	00 +610	+613
	01 +337	+345
	02 -974	-031
120→116	00 +854	+857
	01 +467	+475
	02 -958	-047
160→156	00 P005	Past+1000
	01 +605	
	02 -938	-062
200→195	00 P007	Past +1000
	01 +736	
	02 -914	-086
Final Zero Strains	00 +035	
	01 +005	
	02 -997	-003

$$E_{11} = \frac{(1-\nu_{avg}^2)P}{(\nu_{avg}\epsilon_2+\epsilon_1)A}$$

P_{avg}	$E_{11} \times 10^6$
40	1.225
80	1.320
118	1.390

$E_{11_{avg}} = 1.31 \times 10^6$ psi



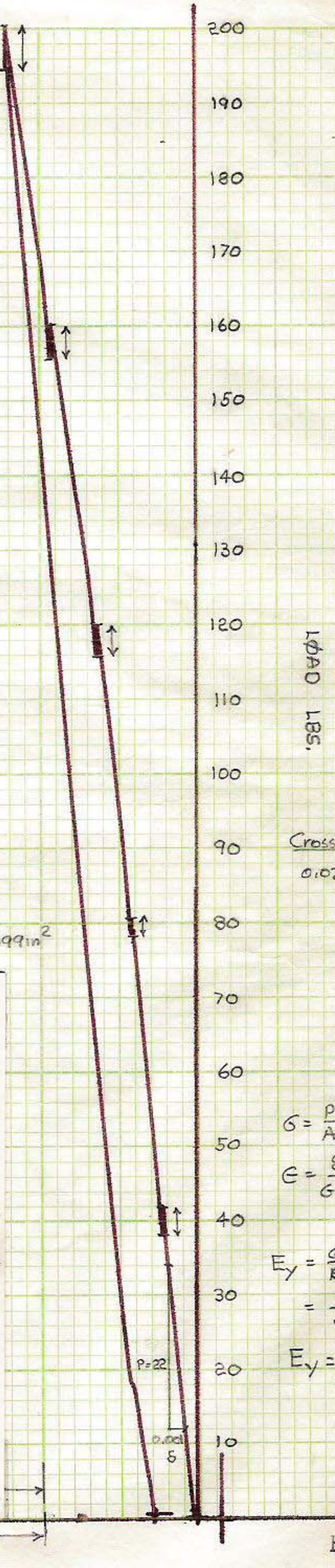
Poisson's ratios

For P_y - $\nu_{12} = |\epsilon_2|/|\epsilon_1|$

For P_x - $\nu_{21} = |\epsilon_1|/|\epsilon_2|$

$\nu_{yx} = \nu_{xy} = \nu_{avg}$

$\nu_{avg} = 0.0583$



Cross Head
0.02 in/min

$\sigma = \frac{P}{A} = \frac{22}{0.099}$

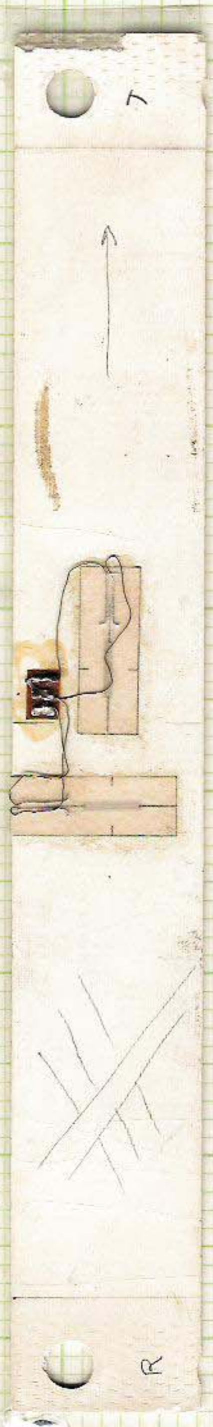
$\epsilon = \frac{\delta}{G.L.} = \frac{0.001}{6}$

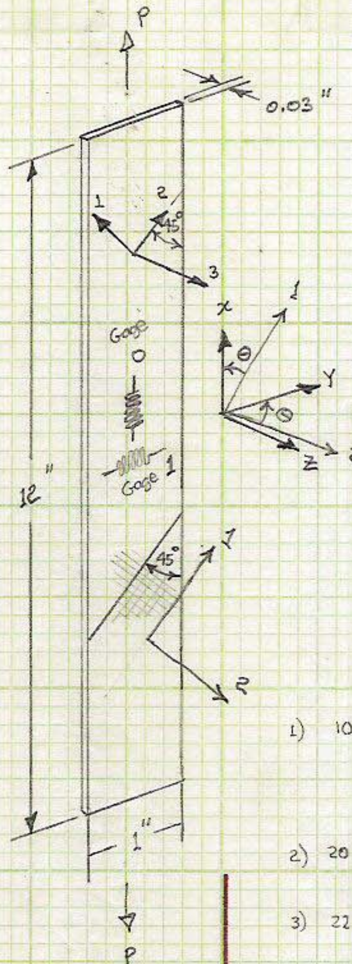
$E_y = \frac{\sigma}{\epsilon}$

$= \frac{132}{99 \times 10^{-5}}$

$E_y = 1.33 \times 10^6$ psi
3

SKIN SHEAR MØDULUS





$$G_{12} = \frac{\sigma_x}{2(\epsilon_x - \epsilon_y)}$$ see reference 4.

initial strains

	00 +003	ϵ_x	0
	01 +005	ϵ_y	0
1) 10 lbs.	00 +354	ϵ_x	351
	01 -883	ϵ_y	22
2) 20 lbs	00 +650	ϵ_x	687
	01 -755	ϵ_y	250
3) 22 lbs	00 +808	ϵ_x	805
	01 -716	ϵ_y	289
4) 24 lbs	00 +887	ϵ_x	890
	01 -686	ϵ_y	319
5) 26 lbs.	00 +949	ϵ_x	952
	01 -659	ϵ_y	346

initial strains

	00 -997	ϵ_x	0
	01 +006	ϵ_y	0
10 lbs	00 +357	ϵ_x	360
	01 -876	ϵ_y	130
20 lbs	00 +705	ϵ_x	708
	01 -750	ϵ_y	256
22 lbs	00 +818	ϵ_x	821
	01 -708	ϵ_y	298
24 lbs	00 +891	ϵ_x	894
	01 -681	ϵ_y	325
26 lbs	00 +909	ϵ_x	972
	01 -651	ϵ_y	355

Run 2.	Run 1.
G_{12}	G_{12}
1) 5.02×10^5	1) 7.25×10^5
2) 7.60×10^5	2) 7.40×10^5
3) 7.10×0^5	3) 7.00×10^5
4) 7.00×10^5	4) 7.45×10^5
5) 7.1×10^5	5) 7.00×10^5
$G_{12_{avg}} = 6.76 \times 10^5$ psi	$G_{12_{avg}} = 7.22 \times 10^5$ psi

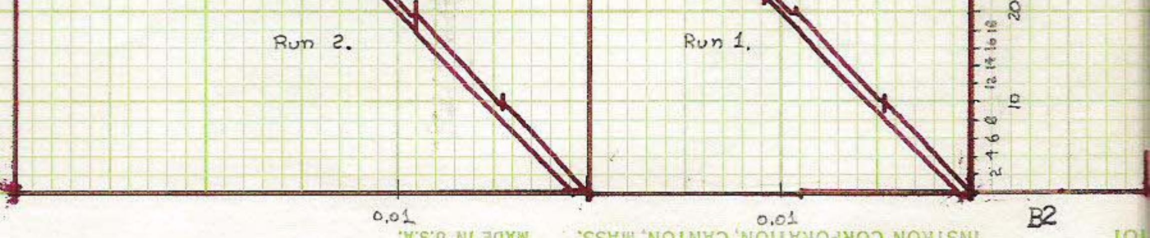
Best

$$E = \frac{P/A}{\Delta L/L} = \frac{20/0.03}{0.009/12}$$

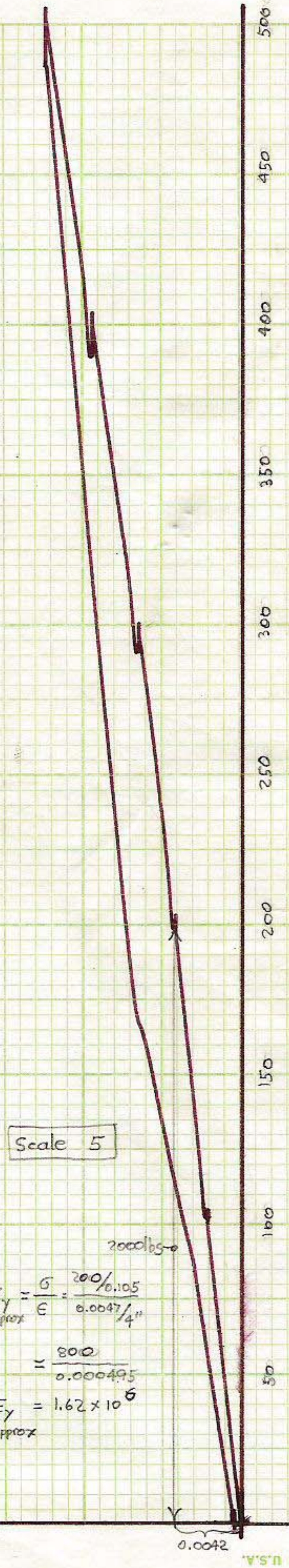
$E = 240/.00027$

$E = 8.9 \times 10^5$ psi $45°$

$E = 8.9 \times 10^5$ psi $45°$



SPAR CAP MODULUS



	Gage No.	Strains $\mu^{in}/_{in}$	
initial 0 lbs	00	+000	
	01	−999	
Average Load lbs			
1050	00	−938 x	−26
	01	+168 y	+169
2000	00	−871 x	−129
	01	+333 y	+334
2900	00	−808 x	−192
	01	+494 y	+495
4000	00	−739 x	−261
	01	+667 y	+668
4900	00	−670 x	−330
	01	+840 y	+841
Final 0 lbs.	00	+000	
	01	+001	

	Gage No.	Strains $\mu^{in}/_{m}$	
initial 0 lbs	00	−997	
	01	−999	
Average LOAD Lbs.			
2000	00	−869	−128
	01	+330	+331
5000	00	−672	−325
	01	+840	+841
7000 over Scale	00	−550	
	01	P007	
Final 0 lbs.	00	−980	
	01	+020	

$\nu_{yx} = |\epsilon_x| / |\epsilon_y|$

$\nu_{yx_{avg}} = 0.342$

$$E_y = \frac{(1-\nu_{avg}^2)P}{(\nu_{avg}\epsilon_x - \epsilon_y)A}$$

P_{avg}	$E_y \times 10^6$
105.	5.515
200.	5.802
290.	5.618
400.	5.813
490.	5.659

$E_{y_{avg}} = 5.739 \times 10^6$ psi

$E_{y_{approx}} = \frac{\sigma}{\epsilon} = \frac{200/0.105}{0.0037/4}$

$= \frac{800}{4.2\times10^{-4}}$

$E_{y_{approx}} = 1.9 \times 10^6$

Cross head speed

0.05 in/min

Drive BY Low BX FWD

Scale 20

Scale 5

Chart speed

5 in/min

Drive BY High BX

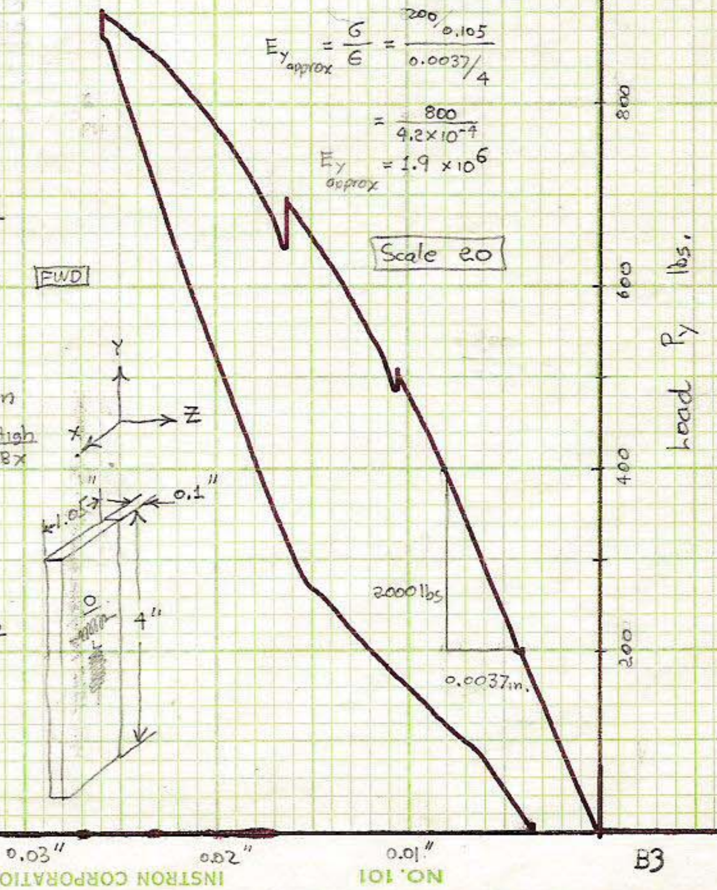
$E_{y_{approx}} = \frac{\sigma}{\epsilon} = \frac{200/0.105}{0.0047/4''}$

$\simeq \frac{800}{0.000495}$

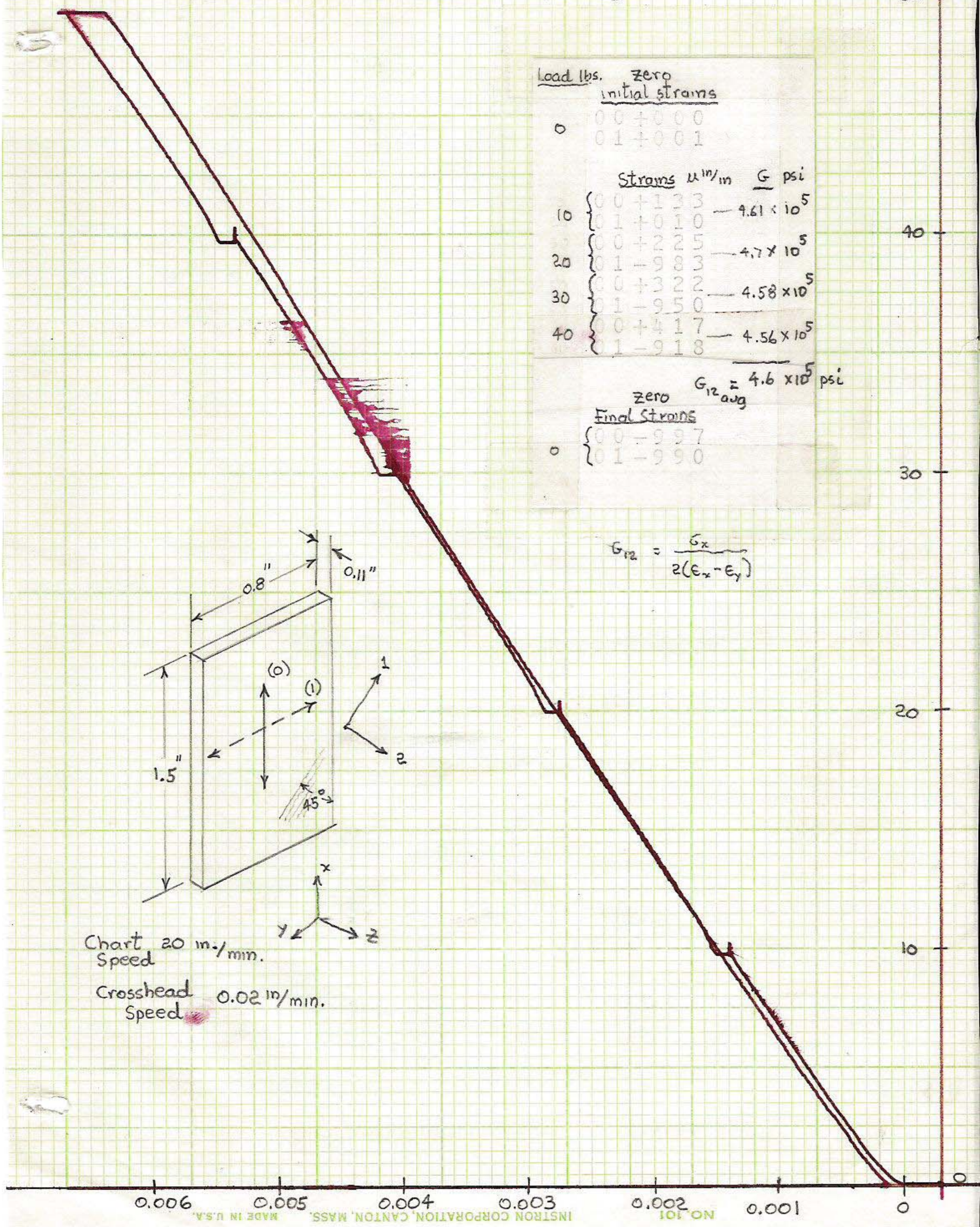
$E_{y_{approx}} = 1.62 \times 10^6$

Specimen Dimensions

Crossection Area = 0.105 in^2



SPAR CAP SHEAR MØDULUS



Load lbs.	Strains $\mu^{in}/_{in}$	G psi
	zero initial strains	
0	00 +000 01 +001	
10	00 +133 01 +010	4.61 × 10^5
20	00 +225 01 −983	4.7 × 10^5
30	00 +322 01 −950	4.58 × 10^5
40	00 +417 01 −918	4.56 × 10^5
	zero Final Strains	
0	00 −997 01 −990	

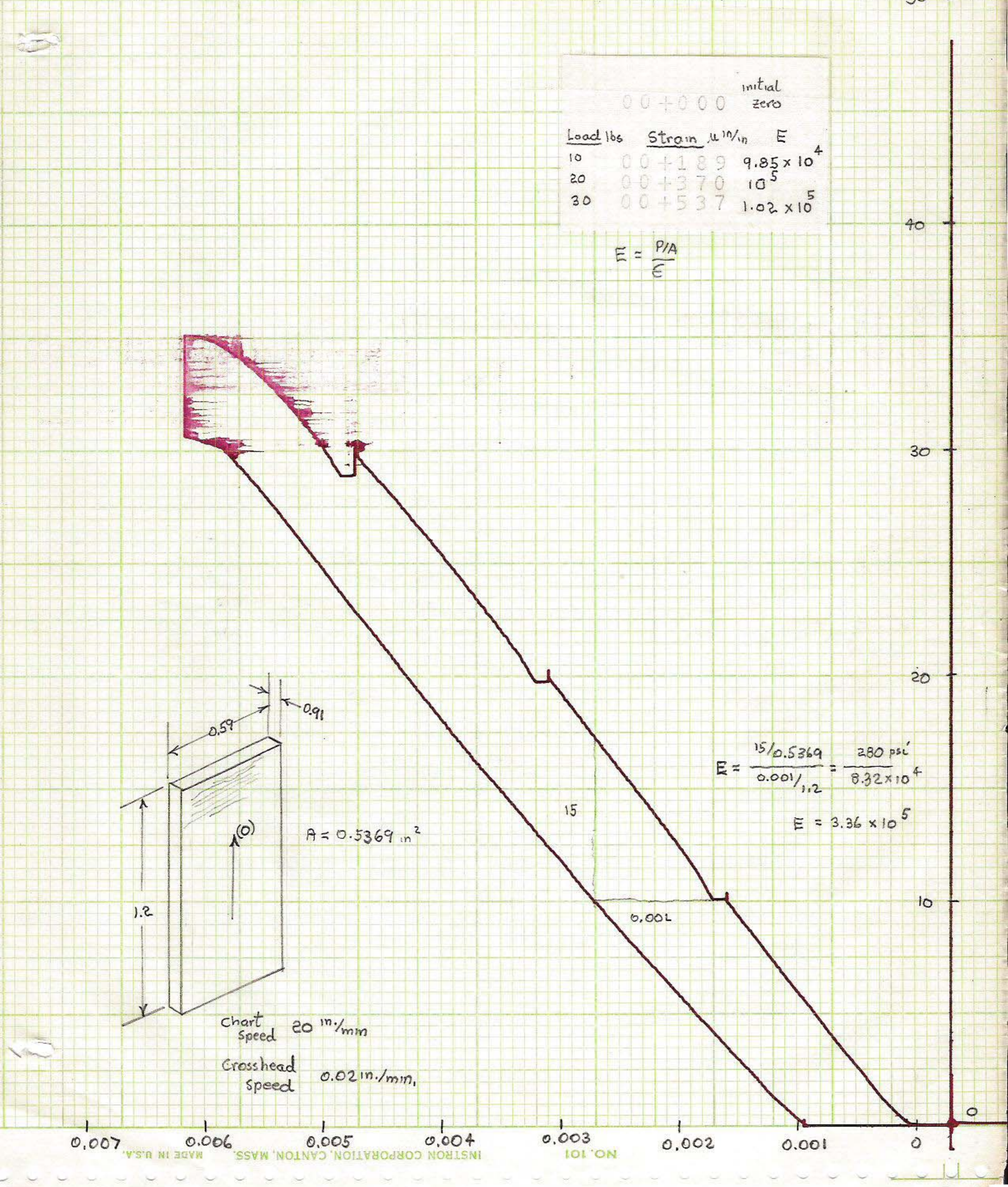
$G_{12_{avg}} = 4.6 \times 10^5$ psi

$$G_{12} = \frac{\sigma_x}{2(\epsilon_x - \epsilon_y)}$$

Chart Speed 20 in./min.

Crosshead Speed 0.02 in/min.

SPAR CAP TRANSVERSE MODULUS OF ELASTICITY



00+000 initial zero

Load lbs	Strain μ in/in	E
	00+000	
10	00+189	9.85×10^4
20	00+370	10^5
30	00+537	1.02×10^5

$$E = \frac{P/A}{\epsilon}$$

$$E = \frac{15/0.5369}{0.001/1.2} = \frac{280 \text{ psi}}{8.32 \times 10^4}$$

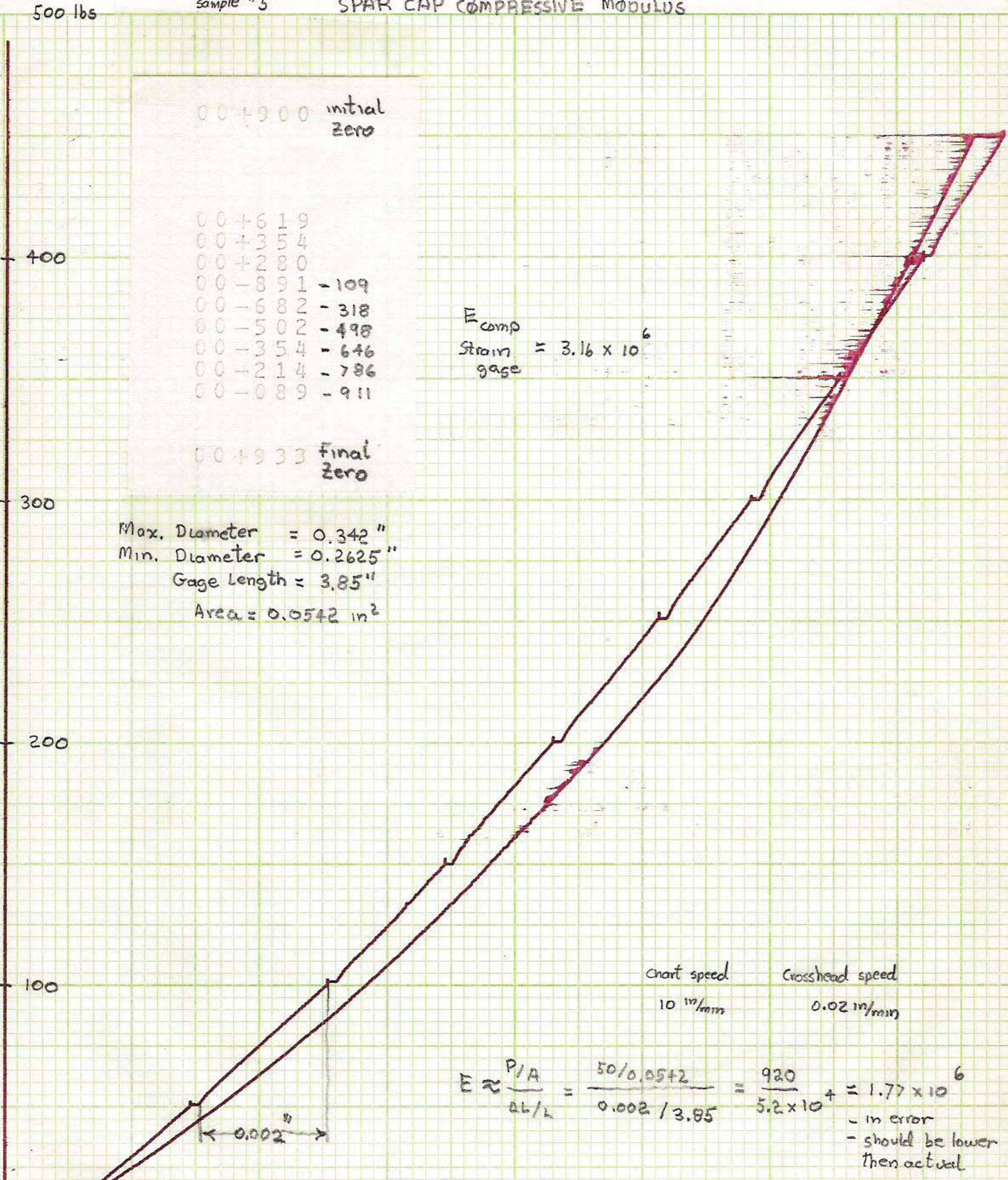
$$E = 3.36 \times 10^5$$

$A = 0.5369 \text{ in}^2$

Chart Speed 20 in./mm

Crosshead speed 0.02 in./min.

sample #5 SPAR CAP COMPRESSIVE MODULUS



00+900 initial Zero

00+619
00+354
00+280
00-891 -109
00-682 -318
00-502 -498
00-354 -646
00-214 -786
00-089 -911

00+933 Final Zero

E_{comp} Strain gage $= 3.16 \times 10^{6}$

Max. Diameter = 0.342"
Min. Diameter = 0.2625"
Gage Length = 3.85"
Area = 0.0542 in²

Chart speed 10 in/min

Crosshead speed 0.02 in/min

$$E \approx \frac{P/A}{\Delta L/L} = \frac{50/0.0542}{0.002/3.85} = \frac{920}{5.2\times10^{4}} = 1.77\times10^{6}$$

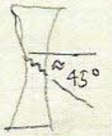
- in error
- should be lower then actual

Compression & Tension | Ultimate Stress

Compression

Sample No.	Area(in^2)	Ultimate Load(lbs)	Ultimate Stress (psi)
1	0.04465	2500	55,950
2	0.0583	3050	51,520
3	0.0358	1440	40,300
4*	0.0382	2920	76,400
5○	0.0542	Strain gage for determining E	
6	0.04935	3150	63,850
Tension			
7	0.0545	3230	59,300
8	0.0510	1640	32,180
9	0.0672	3750	55,800
10	0.0812	4750	58,500
11	0.0553	3450	62,400
12	0.0766	4740	61,850
13	0.0570	1750	30,700
14	0.0589	1800	30,600

* Classic compression fracture at neck of specimen



○ Compression modulus (not corrected for transverse strain) $E = \frac{\sigma}{\epsilon} = \frac{P/A}{\epsilon}$ A = 0.0542

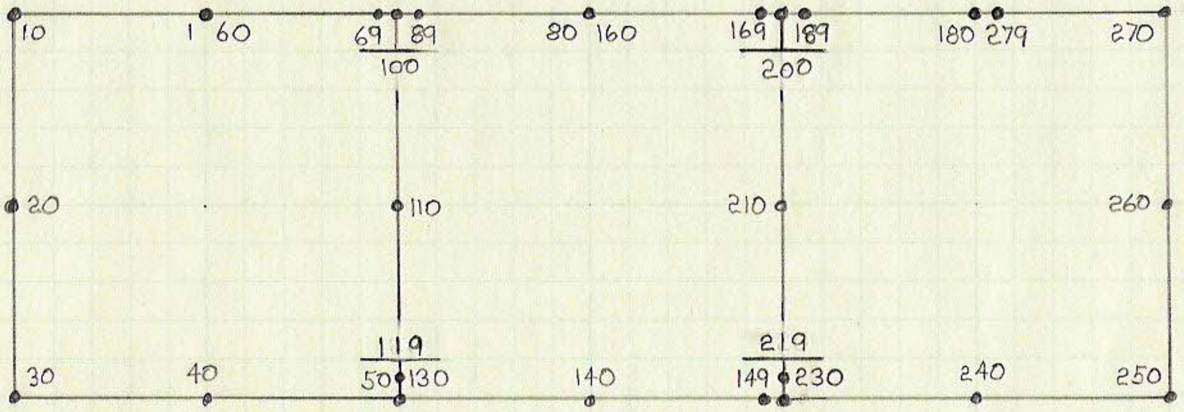
	Point No.	Load, P (lbs.)	Strain, ϵ μin/in	E	
Initial Zero	0	0	0	0	
+900	1	50	291	3.16×10^6	most likely
	2	100	546	3.38×10^6	
	3	150	620	4.45×10^6	
	4	200	1009	3.67×10^6	
	5	250	1218	3.78×10^6	
	6	300	1398	3.95×10^6	
	7	350	1548	4.12×10^6	
	8	400	1686	4.38×10^6	
	9	450	1811	4.59×10^6	

Appendix C

COMPUTER PROGRAM LISTING OF THE STATIC STRESS ANALYSIS OF A TAPERED, MULTICELLED, THIN-WALLED, NONSYMMETRIC CYLINDER

```
30X      DISTANCE BETWEEN TWO SECTIONS#10•0000 = D                                          DY = 10.0
16X      NUMBER OF CELLS#3 = NCELL                                                          NCELL = 3
47X      NUMBER OF POINTS DEFINING SHAPE OF CROSSECTION#29 = NPNTS                          NPNTS = 29
15X      NUMBER OF LEGS# 9 = NLEG                                                           NLEG = 9
22X      CELL 1 HAS A TOTAL OF 3 LEGS•  THE LIST OF LEGS                                    ITLEG(1) = 3
          1 2 4     ASSOCIATED WITH CELL 1                            IARRY(1,1) = 1   IARRY(1,2) = 2   IARRY(1,3) = 4
         CELL 2 HAS A TOTAL OF 5 LEGS•  THE LIST OF LEGS                                    ITLEG(2) = 5
          3 4 5 6 8ASSOCIATED WITH CELL 2                    IARRY(2,1)=3 IARRY(2,2)=4 IARRY(2,3)=5 IARRY(2,4)=6 IARRY(2,5)=8
         CELL 3 HAS A TOTAL OF 3 LEGS•  THE LIST OF LEGS                                    ITLEG(3) = 3
          7 8 9     ASSOCIATED WITH CELL 3                           IARRY(3,1)=7 IARRY(3,2)=8 IARRY(3,3)=9
         •1-1-1     SENSE FOR CELL 1     ISENSE   (1,1) = +   (1,2) = -   (1,3) = -
                                                  (2,1) = +   (2,2) = +   (2,3) = +   (2,4) = -   (2,5) = -
         •1•1•1-1-1SENSE FOR CELL 2               (3,1) = +   (3,2) = +   (3,3) = +
         •1•1•1     SENSE FOR CELL 3                                                   NWEB = 2
30X      NUMBER OF ENCLOSED SHEAR WEBS#2 = NWEB
         LEG  4 FORMS THE SHEAR WEB BETWEEN CELL 1 AND 2                          IARRYW(1) = 4
         LEG  8 FORMS THE SHEAR WEB BETWEEN CELL 2 AND 3                          IARRYW(2) = 8
20X,I2,32X,I2 IN LEG  1 THERE ARE  6 POINTS STARTING WITH POINT NO•  1 AND ENDING WITH  6   LNφ(1) = 6  LBEG(1) = 1
         IN LEG  2 THERE ARE  2 POINTS STARTING WITH POINT NO•  7 AND ENDING WITH  8   LNφ(2) = 2  LBEG(2) = 7
         IN LEG  3 THERE ARE  2 POINTS STARTING WITH POINT NO•  9 AND ENDING WITH 10   LNφ(3) = 2  LBEG(3) = 9
         IN LEG  4 THERE ARE  3 POINTS STARTING WITH POINT NO• 11 AND ENDING WITH 13   LNφ(4) = 3  LBEG(4) = 11
         IN LEG  5 THERE ARE  3 POINTS STARTING WITH POINT NO• 14 AND ENDING WITH 16   LNφ(5) = 3  LBEG(5) = 14
         IN LEG  6 THERE ARE  2 POINTS STARTING WITH POINT NO• 17 AND ENDING WITH 18   LNφ(6) = 2  LBEG(6) = 17
         IN LEG  7 THERE ARE  2 POINTS STARTING WITH POINT NO• 19 AND ENDING WITH 20   LNφ(7) = 2  LBEG(7) = 19
         IN LEG  8 THERE ARE  3 POINTS STARTING WITH POINT NO• 21 AND ENDING WITH 23   LNφ(8) = 3  LBEG(8) = 21
         IN LEG  9 THERE ARE  6 POINTS STARTING WITH POINT NO• 24 AND ENDING WITH 29   LNφ(9) = 6  LBEG(9) = 24
19X,I2,4X,I2 IN LEG  1  Q□ 1*#Q□ 1*•Q□ 1*      IV(1) = 1   IU(1) = 1
         IN LEG  2  Q□ 7*#Q□ 1*•Q□ 1*      IV(2) = 1   IU(2) = 1
         IN LEG  3  Q□ 9*#Q□ 1*•Q□ 1*      IV(3) = 1   IU(3) = 1
         IN LEG  4  Q□11*#Q□ 8*•Q□10*      IV(4) = 8   IU(4) = 10
         IN LEG  5  Q□14*#Q□ 6*•Q□13*      IV(5) = 6   IU(5) = 13
         IN LEG  6  Q□17*#Q□ 1*•Q□ 1*      IV(6) = 1   IU(6) = 1
         IN LEG  7  Q□19*#Q□ 1*•Q□ 1*      IV(7) = 1   IU(7) = 1
         IN LEG  8  Q□21*#Q□18*•Q□20*      IV(8) = 18  IU(8) = 20
         IN LEG  9  Q□24*#Q□16*•Q□23*      IV(9) = 16  IU(9) = 23
          1 10•000  4•000 10•000  4•000  0•100  0•100 •0•250E•07 •0•342E•06
          2 12•000  4•000 12•000  4•000  0•100  0•100 •0•250E•07 •0•342E•06
          3 12•000  2•000 12•000  2•000  0•100  0•100 •0•250E•07 •0•342E•06
          4 12•000  0•000 12•000  0•000  0•100  0•100 •0•250E•07 •0•342E•06
          5 10•000  0•000 10•000  0•000  0•100  0•100 •0•250E•07 •0•342E•06
          6  8•000  0•000  8•000  0•000  0•100  0•100 •0•250E•07 •0•342E•06
          7 10•000  4•000 10•000  4•000  0•100  0•100 •0•250E•07 •0•342E•06
          8  8•000  4•000  8•000  4•000  0•100  0•100 •0•250E•07 •0•342E•06
          9  6•000  4•000  6•000  4•000  0•100  0•100 •0•250E•07 •0•342E•06
          I   RX     RZ     TX     TZ     TR     TT       E          G
```

```
 10  8.000  4.000  8.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 11  8.000  4.000  8.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 12  8.000  2.000  8.000  2.000  0.100  0.100  .0.250E.07  .0.342E.06
 13  8.000  0.000  8.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 14  8.000  0.000  8.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 15  6.000  0.000  6.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 16  4.000  0.000  4.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 17  6.000  4.000  6.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 18  4.000  4.000  4.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 19  2.000  4.000  2.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 20  4.000  4.000  4.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 21  4.000  4.000  4.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 22  4.000  2.000  4.000  2.000  0.100  0.100  .0.250E.07  .0.342E.06
 23  4.000  0.000  4.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 24  4.000  0.000  4.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 25  2.000  0.000  2.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 26  0.000  0.000  0.000  0.000  0.100  0.100  .0.250E.07  .0.342E.06
 27  0.000  2.000  0.000  2.000  0.100  0.100  .0.250E.07  .0.342E.06
 28  0.000  4.000  0.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
 29  2.000  4.000  2.000  4.000  0.100  0.100  .0.250E.07  .0.342E.06
YF  #   0.0      YL# 100.0
RXAC#   6.0    RZAC#   2.0    TXAC#   6.0    TZAC#   2.0
SKIN DENSITY LBS./IN.3 #.0.250E-01  = RHφS
CORE DENSITY LBS./IN.3 #.0.100E-02  = RHφC
CORE THICKNESS IN.#.0.100E.00  = DC
ANGLE OF ATTACK# 0.00000     IN DEGREES  NO INCIDINCE ANGLE     ATANGD= 0.0
CHORD ANGLE# 0.00000    IN DEGREES                             CHAND = 0.0
SKIN@S BENDING MODULUS#.0.250E.07 LB/IN**2                      ES = 0.250E+07
THE NUMBER OF SECTIONS WHERE WRITEOUT IS DESIRED# 2             NN = 2
INFORMATION        ETC. AT SECTION Y# 00.00                     YS(1) = 0.0
INFORMATION        ETC. AT SECTION Y# 50.00                     YS(2) = 50.0
```


10
1 60
69 | 89
100
80 160
169 | 189
200
180 279
270
20
110
210
260
119
219
30
40
50 130
140
149 230
240
250

```
      WING-DIMENSIONS,PROPERTIES, AND PRINT OUT LOCATIONS
THE FINAL WING SECTION IS EVALUATED AT Y=100.0000
DISTANCE BETWEEN TWO SECTIONS=10.0000
NUMBER OF CELLS=3
NUMBER OF POINTS DEFINING SHAPE OF CROSSECTION=29
NUMBER OF LEGS= 9
CELL 1 HAS A TOTAL OF 3 LEGS.  THE LIST OF LEGS
124  ASSOCIATED WITH CELL 1
CELL 2 HAS A TOTAL OF 5 LEGS.  THE LIST OF LEGS
34568ASSOCIATED WITH CELL 2
CELL 3 HAS A TOTAL OF 3 LEGS.  THE LIST OF LEGS
789  ASSOCIATED WITH CELL 3
.1-1-1     SENSE FOR CELL 1
.1.1.1-1-1SENSE FOR CELL 2
.1.1.1     SENSE FOR CELL 3
NUMBER OF ENCLOSED SHEAR WEBS=2
LEG  4 FORMS THE SHEAR WEB BETWEEN CELL 1 AND 2
LEG  8 FORMS THE SHEAR WEB BETWEEN CELL 2 AND 3
IN LEG  1 THERE ARE  6 POINTS STARTING WITH POINT NO.  1 AND ENDING WITH  6
IN LEG  2 THERE ARE  2 POINTS STARTING WITH POINT NO.  7 AND ENDING WITH  8
IN LEG  3 THERE ARE  2 POINTS STARTING WITH POINT NO.  9 AND ENDING WITH 10
IN LEG  4 THERE ARE  3 POINTS STARTING WITH POINT NO. 11 AND ENDING WITH 13
IN LEG  5 THERE ARE  3 POINTS STARTING WITH POINT NO. 14 AND ENDING WITH 16
IN LEG  6 THERE ARE  2 POINTS STARTING WITH POINT NO. 17 AND ENDING WITH 18
IN LEG  7 THERE ARE  2 POINTS STARTING WITH POINT NO. 19 AND ENDING WITH 20
IN LEG  8 THERE ARE  3 POINTS STARTING WITH POINT NO. 21 AND ENDING WITH 23
IN LEG  9 THERE ARE  6 POINTS STARTING WITH POINT NO. 24 AND ENDING WITH 29
IN LEG  1  Q) 1*=Q) 1*.Q) 1*
IN LEG  2  Q)14*=Q) 1*.Q) 1*
IN LEG  3  Q)18*=Q) 1*.Q) 1*
IN LEG  4  Q)11*=Q) 8*.Q)10*
IN LEG  5  Q)14*=Q) 6*.Q)13*
IN LEG  6  Q)33*=Q) 1*.Q) 1*
IN LEG  7  Q)37*=Q) 1*.Q) 1*
IN LEG  8  Q)21*=Q)18*.Q)20*
IN LEG  9  Q)24*=Q)16*.Q)23*
RX 1=10.000RZ 1= 4.000TX 1=10.000TZ 1= 4.000RT 1= 0.100TT 1= 0.100
RX 2=12.000RZ 2= 4.000TX 2=12.000TZ 2= 4.000RT 2= 0.100TT 2= 0.100
RX 3=12.000RZ 3= 2.000TX 3=12.000TZ 3= 2.000RT 3= 0.100TT 3= 0.100
RX 4=12.000RZ 4=00.000TX 3=12.000TZ 4=00.000RT 4= 0.100TT 4= 0.100
RX 5=10.000RZ 5=00.000TX 5=10.000TZ 5=00.000RT 5= 0.100TT 5= 0.100
RX 6= 8.000RZ 6=00.000TX 6= 8.000TZ 6=00.000RT 6= 0.100TT 6= 0.100
RX 7=10.000RZ 7= 4.000TX 7=10.000TZ 7= 4.000RT 7= 0.100TT 7= 0.100
RX 8= 8.000RZ 8= 4.000TX 8= 8.000TZ 8= 4.000RT 8= 0.100TT 8= 0.100
RX 9= 6.000RZ 9= 4.000TX 9= 6.000TZ 9= 4.000RT 9= 0.100TT 9= 0.100
RX10= 8.000RZ10= 4.000TX10= 8.000TZ10= 4.000RT10= 0.100TT10= 0.100
RX11= 8.000RZ11= 4.000TX11= 8.000TZ11= 4.000RT11= 0.100TT11= 0.100
RX12= 8.000RZ12= 2.000TX12= 8.000TZ12= 2.000RT12= 0.100TT12= 0.100
RX13= 8.000RZ13=00.000TX13= 8.000TZ13=00.000RT13= 0.100TT13= 0.100
RX14= 8.000RZ14=00.000TX14= 8.000TZ14=00.000RT14= 0.100TT14= 0.100
RX15= 6.000RZ15=00.000TX15= 6.000TZ15=00.000RT15= 0.100TT15= 0.100
RX16= 4.000RZ16=00.000TX16= 4.000TZ16=00.000RT16= 0.100TT16= 0.100
RX17= 6.000RZ17= 4.000TX17= 6.000TZ17= 4.000RT17= 0.100TT17= 0.100
RX18= 4.000RZ18= 4.000TX18= 4.000TZ18= 4.000RT18= 0.100TT18= 0.100
RX19= 2.000RZ19= 4.000TX19= 2.000TZ19= 4.000RT19= 0.100TT19= 0.100
RX20= 4.000RZ20= 4.000TX20= 4.000TZ20= 4.000RT20= 0.100TT20= 0.100
RX21= 4.000RZ21= 4.000TX21= 4.000TZ21= 4.000RT21= 0.100TT21= 0.100
RX22= 4.000RZ22= 2.000TX22= 4.000TZ22= 2.000RT22= 0.100TT22= 0.100
RX23= 4.000RZ23=00.000TX23= 4.000TZ23=00.000RT23= 0.100TT23= 0.100
RX24= 4.000RZ24=00.000TX24= 4.000TZ24=00.000RT24= 0.100TT24= 0.100
```

```
RX25= 2.000RZ25=00.000TX25= 2.000TZ25=00.000RT25= 0.100TT25= 0.100
RX26=00.000RZ26=00.000TX26=00.000TZ26=00.000RT26= 0.100TT26= 0.100
RX27=00.000RZ27= 2.000TX27=00.000TZ27= 2.000RT27= 0.100TT27= 0.100
RX28=00.000RZ28= 4.000TX28=00.000TZ28= 4.000RT28= 0.100TT28= 0.100
RX29= 2.000RZ29= 4.000TX29= 2.000TZ29= 4.000RT29= 0.100TT29= 0.100
YF  =   0.0       YL= 100.0
RXAC=   6.0    RZAC=   2.0     TXAC=   6.0    TZAC=   2.0
SKIN DENSITY LBS./IN.3 =.0.250E-01
CORE DENSITY LBS./IN.3 =.0.100E-02
CORE THICKNESS IN.=.0.100E.00
ANGLE OF ATTACK= 0.00000      IN DEGREES  NO INCIDINCE ANGLE
CHORD ANGLE= 0.00000   IN DEGREES
MODULUS OF ELASTICITY=.0.250E.07 LB/IN**2
SHEAR MODULUS=.0.342E.06 LB/IN**2
THE NUMBER OF SECTIONS WHERE WRITEOUT IS DESIRED= 2
INFORMATION         ETC. AT SECTION Y= 00.00
INFORMATION         ETC. AT SECTION Y= 50.00
```

Appendix D

COMPUTER PROGRAM LISTING OF THE STRESS-STRAIN RELATIONSHIPS OF THIN ORTHOTROPIC LAMINATED PLATES IN PLANE STRESS

```
      DIMENSION Q(6,6),QB(6,6),T(6,6),S(3,3),SIGMA(6),SIGMAT(6),STRN(6)
      NDEG=19
      READ(5,10)NUMBER
      READ(5,20)E11,E22,V12,G
      V21=V12*E22/E11
      WRITE(6,40)NUMBER,E11,E22,V12,V21,G
      DO 2 I=1,6
      DO 2 J=1,6
      Q(I,J)=0.0
      QB(I,J)=0.0
      T(I,J)=0.0
      STRN(I)=0.0
      SIGMA(I)=0.0
      SIGMAT(I)=0.0
    2 CONTINUE
      Q(1,1)=E11/(1.-V12*V21)
      Q(2,2)=E22/(1.-V12*V21)
      Q(6,6)=G
      Q(1,2)=V21*E11/(1.-V12*V21)
      Q(2,1)=Q(1,2)
      S(1,1)=1./E11
      S(1,2)=-V12/E11
      S(1,3)=0.0
      S(2,1)=-V21/E22
      S(2,2)=1./E22
      S(2,3)=0.0
      S(3,1)=0.0
      S(3,2)=0.0
      S(3,3)=1./G
      U1=(3.*Q(1,1)+3.*Q(2,2)+2.*Q(1,2)+4.*Q(6,6))/8.
      U2=(Q(1,1)-Q(2,2))/2.
      U3=(Q(1,1)+Q(2,2)-2.*Q(1,2)-4.*Q(6,6))/8.
      U4=(Q(1,1)+Q(2,2)+6.*Q(1,2)-4.*Q(6,6))/8.
      U5=(Q(1,1)+Q(2,2)-2.*Q(1,2)+4.*Q(6,6))/8.
      WRITE(6,54)U1,U2,U3,U4,U5
      THETAD=0.0
      DO 4 I=1,NDEG
      THETA=THETAD*2.*3.1416/360.
      THETA2=2.*THETA
      THETA4=4.*THETA
      RN=SIN(THETA)
      RM=COS(THETA)
      QB(1,1)=U1+U2*COS(THETA2)+U3*COS(THETA4)
      QB(2,2)=U1-U2*COS(THETA2)+U3*COS(THETA4)
      QB(1,2)=U4-U3*COS(THETA4)
      QB(6,6)=U5-U3*COS(THETA4)
      QB(1,6)=-0.5*U2*SIN(THETA2)-U3*SIN(THETA4)
      QB(2,6)=-0.5*U2*SIN(THETA2)+U3*SIN(THETA4)
      QB(2,1)=QB(1,2)
      QB(6,1)=QB(1,6)
      QB(6,2)=QB(2,6)
      EXX=1./(RM**4*S(1,1)+2.*RM**2*RN**2*S(1,2)+2.*RM**3*RN*S(1,3)+RN**
     *4*S(2,2)+2.*RM*RN**3*S(2,3)+RM**2*RN**2*S(3,3))
      EYY=1./(RN**4*S(1,1)+2.*RM**2*RN**2*S(1,2)-2.*RM*RN**3*S(1,3)+RM**
     *4*S(2,2)-2.*RM**3*RN*S(2,3)+RM**2*RN**2*S(3,3))
      GXY=1./(4.*RM**2*RN**2*S(1,1)-8.*RM**2*RN**2*S(1,2)+(4.*RM*RN**3-4
     *.*RN*RM**3)*S(1,3)+4.*RM**2*RN**2*S(2,2)+(4.*RN*RM**3-4.*RM*RN**3)
     **S(2,3)+(RM**2-RN**2)**2*S(3,3))
```

```
      WRITE(6,55)THETAD,QB(1,1),QB(2,2),QB(6,6),QB(1,2),QB(1,6),QB(2,6)
       WRITE(6,50)EXX,EYY,GXY
      THETAD=THETAD+5.
    4 CONTINUE
      WRITE(6,90)
      DO 500 ICOUNT=1,NUMBER
      READ(5,30)STRN1,STRN2,STRN12,THETAD
      STRN(1)=STRN1*1.0E-6
      STRN(2)=STRN2*1.0E-6
      STRN(6)=(+2.*STRN12-STRN1-STRN2)*1.0E-6
      THETA=THETAD*2.*3.1416/360.
      RN=SIN(THETA)
      RM=COS(THETA)
      EXX=1./(RM**4*S(1,1)+2.*RM**2*RN**2*S(1,2)+2.*RM**3*RN*S(1,3)+RN**
     *4*S(2,2)+2.*RM*RN**3*S(2,3)+RM**2*RN**2*S(3,3))
      EYY=1./(RN**4*S(1,1)+2.*RM**2*RN**2*S(1,2)-2.*RM*RN**3*S(1,3)+RM**
     *4*S(2,2)-2.*RM**3*RN*S(2,3)+RM**2*RN**2*S(3,3))
      GXY=1./(4.*RM**2*RN**2*S(1,1)-8.*RM**2*RN**2*S(1,2)+(4.*RM*RN**3-4
     *.*RN*RM**3)*S(1,3)+4.*RM**2*RN**2*S(2,2)+(4.*RN*RM**3-4.*RM*RN**3)
     **S(2,3)+(RM**2-RN**2)**2*S(3,3))
      T(1,1)=(COS(THETA))**2
      T(1,2)=(SIN(THETA))**2
      T(1,6)=2.*SIN(THETA)*COS(THETA)
      T(2,1)=T(1,2)
      T(2,2)=T(1,1)
      T(2,6)=-T(1,6)
      T(6,1)=-SIN(THETA)*COS(THETA)
      T(6,2)=-T(6,1)
      T(6,6)=T(1,1)-T(1,2)
      DO 200 I=1,6
      SIGM=0.
      DO 100 J=1,6
      SIGM=Q(I,J)*STRN(J)+SIGM
  100 CONTINUE
      SIGMA(I)=SIGM
  200 CONTINUE
      DO 400 I=1,6
      SIGMT=0.
      DO 300 J=1,6
      SIGMT=T(I,J)*SIGMA(J)+SIGMT
  300 CONTINUE
      SIGMAT(I)=SIGMT
  400 CONTINUE
      WRITE(6,80)ICOUNT
      WRITE(6,60)STRN1,STRN2,STRN12,THETAD
      WRITE(6,70)SIGMAT(2),SIGMAT(6)
      WRITE(6,50)EXX,EYY,GXY
  500 CONTINUE
   10 FORMAT(29X,I2)
   20 FORMAT(48X,E10.3,/,53X,E10.3,/,54X,E10.3,/,45X,E10.3)
   30 FORMAT(41X,E10.3,/,44X,E10.3,/,44X,E10.3,/,71X,F7.3)
   40 FORMAT(1X,'TOTAL NUMBER OF STRAIN GAGES=',I2,/,' MODULUS OF ELASTI
     *CITY PARALLEL TO LAMINATE AXIS=',E10.3,/,' MODULUS OF ELASTICITY P
     *ERPENDICULAR TO LAMINATE AXIS=',E10.3,/,' POISSONS RATIO WITH THE
     *LOAD ALONG THE LAMINATE AXIS=',E10.3,/,' POISSONS RATIO WITH THE L
     *OAD PERPENDICULAR TO LAMINATE AXIS=',E10.3,' SHEAR MODULUS=',E10.3
     *)
   50 FORMAT(1X,'EXX=',E15.8,/,' EYY=',E15.8,/,' GXY=',E15.8)
```

```
54 FORMAT(1H1,'U1=',E15.8,/,' U2=',E15.8,/,' U3=',E15.8,/,' U4=',E15.
  *8,/,' U5=',E15.8)
55 FORMAT(1X,'THETA=',F7.3,/,' QB(1,1)=',E15.8,/,' QB(2,2)=',E15.8,/,
  *' QB(6,6)=',E15.8,/,' QB(1,2)=',E15.8,/,' QB(1,6)=',E15.8,/,' QB(2
  *,6)=',E15.8)
60 FORMAT(1X,'STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS=',E10.3,/,' ST
  *RAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS=',E10.3,/,' STRAIN IN MI
  *CRO-IN./IN. 45DEG TO LAMINATE AXIS=',E10.3,/,' ANGLE OF ROTATION F
  *ROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS=',F7.3)
70 FORMAT(1X,'NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST=',E10.3,/,'
  * SHEAR STRESS AT SAME PLANE=',E10.3)
80 FORMAT(///,18X,'PROPERTIES AT GAGE NO.',I2)
90 FORMAT(1H1)
   CALL EXIT
   END
```

```
U1= 0.13660080E 07
U2= 0.0
U3=-0.51541625E 05
U4= 0.25091625E 05
U5= 0.67045838E 06
THETA=  0.0
QB(1,1)= 0.13144660E 07
QB(2,2)= 0.13144660E 07
QB(6,6)= 0.72200000E 06
QB(1,2)= 0.76633250E 05
QB(1,6)= 0.0
QB(2,6)= 0.0
EXX= 0.13100000E 07
EYY= 0.13100000E 07
GXY= 0.72200013E 06
THETA=  5.000
QB(1,1)= 0.13175740E 07
QB(2,2)= 0.13175740E 07
QB(6,6)= 0.71889163E 06
QB(1,2)= 0.73524875E 05
QB(1,6)= 0.17628301E 05
QB(2,6)=-0.17628301E 05
EXX= 0.13129910E 07
EYY= 0.13129910E 07
GXY= 0.71839275E 06
THETA= 10.000
QB(1,1)= 0.13265240E 07
QB(2,2)= 0.13265240E 07
QB(6,6)= 0.70994150E 06
QB(1,2)= 0.64574758E 05
QB(1,6)= 0.33130363E 05
QB(2,6)=-0.33130363E 05
EXX= 0.13216800E 07
EYY= 0.13216800E 07
GXY= 0.70820269E 06
THETA= 15.000
QB(1,1)= 0.13402370E 07
QB(2,2)= 0.13402370E 07
QB(6,6)= 0.69622906E 06
QB(1,2)= 0.50862359E 05
QB(1,6)= 0.44636398E 05
QB(2,6)=-0.44636398E 05
EXX= 0.13352180E 07
EYY= 0.13352180E 07
GXY= 0.69313925E 06
THETA= 20.000
QB(1,1)= 0.13570580E 07
QB(2,2)= 0.13570580E 07
QB(6,6)= 0.67940831E 06
QB(1,2)= 0.34041602E 05
QB(1,6)= 0.50758617E 05
QB(2,6)=-0.50758617E 05
EXX= 0.13522090E 07
EYY= 0.13522090E 07
GXY= 0.67551425E 06
THETA= 25.000
QB(1,1)= 0.13749580E 07
QB(2,2)= 0.13749580E 07
QB(6,6)= 0.66150806E 06
QB(1,2)= 0.16141344E 05
QB(1,6)= 0.50758563E 05
QB(2,6)=-0.50758563E 05
EXX= 0.13707720E 07
EYY= 0.13707720E 07
GXY= 0.65771669E 06
THETA= 30.000
QB(1,1)= 0.13917790E 07
QB(2,2)= 0.13917790E 07
QB(6,6)= 0.64468738E 06
QB(1,2)=-0.67937500E 03
QB(1,6)= 0.44636238E 05
QB(2,6)=-0.44636238E 05
EXX= 0.13886860E 07
EYY= 0.13886860E 07
GXY= 0.64182681E 06
THETA= 35.000
QB(1,1)= 0.14054910E 07
QB(2,2)= 0.14054910E 07
QB(6,6)= 0.63097500E 06
QB(1,2)=-0.14391723E 05
QB(1,6)= 0.33130109E 05
QB(2,6)=-0.33130109E 05
EXX= 0.14036390E 07
EYY= 0.14036390E 07
GXY= 0.62943000E 06
THETA= 40.000
QB(1,1)= 0.14144410E 07
QB(2,2)= 0.14144410E 07
QB(6,6)= 0.62202494E 06
QB(1,2)=-0.23341770E 05
QB(1,6)= 0.17627973E 05
QB(2,6)=-0.17627973E 05
EXX= 0.14135740E 07
EYY= 0.14135740E 07
GXY= 0.62159369E 06
THETA= 45.000
QB(1,1)= 0.14175490E 07
QB(2,2)= 0.14175490E 07
QB(6,6)= 0.61891675E 06
QB(1,2)=-0.26450000E 05
QB(1,6)=-0.31171787E 00
QB(2,6)= 0.31171787E 00
EXX= 0.14170570E 07
EYY= 0.14170570E 07
GXY= 0.61891769E 06
THETA= 50.000
QB(1,1)= 0.14144410E 07
QB(2,2)= 0.14144410E 07
QB(6,6)= 0.62202519E 06
QB(1,2)=-0.23341539E 05
QB(1,6)=-0.17628605E 05
QB(2,6)= 0.17628605E 05
EXX= 0.14135740E 07
EYY= 0.14135740E 07
GXY= 0.62159400E 06
THETA= 55.000
QB(1,1)= 0.14054900E 07
QB(2,2)= 0.14054900E 07
QB(6,6)= 0.63097544E 06
QB(1,2)=-0.14391293E 05
QB(1,6)=-0.33130621E 05
QB(2,6)= 0.33130621E 05
EXX= 0.14036380E 07
EYY= 0.14036380E 07
GXY= 0.62943031E 06
THETA= 60.000
QB(1,1)= 0.13917780E 07
QB(2,2)= 0.13917780E 07
QB(6,6)= 0.64468781E 06
QB(1,2)=-0.67887891E 03
QB(1,6)=-0.44636527E 05
QB(2,6)= 0.44636527E 05
EXX= 0.13886850E 07
EYY= 0.13886850E 07
GXY= 0.64182719E 06
THETA= 65.000
QB(1,1)= 0.13749570E 07
QB(2,2)= 0.13749570E 07
QB(6,6)= 0.66150850E 06
QB(1,2)= 0.16141762E 05
QB(1,6)=-0.50758637E 05
QB(2,6)= 0.50758637E 05
EXX= 0.13707720E 07
EYY= 0.13707720E 07
GXY= 0.65771706E 06
THETA= 70.000
QB(1,1)= 0.13570570E 07
QB(2,2)= 0.13570570E 07
QB(6,6)= 0.67940881E 06
QB(1,2)= 0.34042066E 05
QB(1,6)=-0.50758535E 05
QB(2,6)= 0.50758535E 05
EXX= 0.13522090E 07
EYY= 0.13522090E 07
GXY= 0.67551469E 06
THETA= 75.000
QB(1,1)= 0.13402360E 07
QB(2,2)= 0.13402360E 07
QB(6,6)= 0.69622956E 06
QB(1,2)= 0.50862813E 05
QB(1,6)=-0.44636133E 05
QB(2,6)= 0.44636133E 05
EXX= 0.13352180E 07
EYY= 0.13352180E 07
GXY= 0.69313969E 06
THETA= 80.000
QB(1,1)= 0.13265240E 07
QB(2,2)= 0.13265240E 07
QB(6,6)= 0.70994188E 06
QB(1,2)= 0.64575141E 05
QB(1,6)=-0.33129906E 05
QB(2,6)= 0.33129906E 05
EXX= 0.13216800E 07
EYY= 0.13216800E 07
GXY= 0.70820313E 06
THETA= 85.000
QB(1,1)= 0.13175740E 07
QB(2,2)= 0.13175740E 07
QB(6,6)= 0.71889175E 06
QB(1,2)= 0.73525000E 05
QB(1,6)=-0.17627867E 05
QB(2,6)= 0.17627867E 05
EXX= 0.13129910E 07
EYY= 0.13129910E 07
GXY= 0.71839319E 06
THETA= 90.000
QB(1,1)= 0.13144660E 07
QB(2,2)= 0.13144660E 07
QB(6,6)= 0.72200000E 06
QB(1,2)= 0.76633250E 05
QB(1,6)= 0.52512199E 00
QB(2,6)=-0.52512199E 00
EXX= 0.13100000E 07
EYY= 0.13100000E 07
GXY= 0.72200013E 06
```

```
                    PROPERTIES AT GAGE NO. 1
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.200E 03
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS=-0.170E 02
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS=-0.263E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 41.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST= 0.616E 03
SHEAR STRESS AT SAME PLANE=-0.204E 03
EXX= 0.14148180E 07
EYY= 0.14148180E 07
GXY= 0.62063406E 06


                    PROPERTIES AT GAGE NO. 2
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.217E 03
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS= 0.520E 02
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS=-0.220E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 40.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST= 0.673E 03
SHEAR STRESS AT SAME PLANE=-0.189E 03
EXX= 0.14135740E 07
EYY= 0.14135740E 07
GXY= 0.62159369E 06


                    PROPERTIES AT GAGE NO. 3
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.165E 03
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS= 0.160E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS=-0.194E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 40.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST= 0.732E 03
SHEAR STRESS AT SAME PLANE=-0.924E 02
EXX= 0.14135740E 07
EYY= 0.14135740E 07
GXY= 0.62159369E 06


                    PROPERTIES AT GAGE NO. 4
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.390E 02
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS= 0.232E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS=-0.222E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 31.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST= 0.700E 03
SHEAR STRESS AT SAME PLANE=-0.137E 03
EXX= 0.13919850E 07
EYY= 0.13919850E 07
GXY= 0.63902650E 06


                    PROPERTIES AT GAGE NO. 5
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.680E 02
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS= 0.131E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS=-0.281E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 37.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST= 0.677E 03
SHEAR STRESS AT SAME PLANE=-0.114E 03
EXX= 0.14083130E 07
EYY= 0.14083130E 07
```

```
GXY= 0.62570475E 06


               PROPERTIES AT GAGE NO. 6
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS=-0.148E 03
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS=-0.158E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS= 0.357E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 48.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST=-0.945E 03
SHEAR STRESS AT SAME PLANE=-0.831E 02
EXX= 0.14157920E 07
EYY= 0.14157920E 07
GXY= 0.61988456E 06


               PROPERTIES AT GAGE NO. 7
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS= 0.520E 02
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS=-0.209E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS= 0.249E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 45.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST=-0.582E 03
SHEAR STRESS AT SAME PLANE=-0.162E 03
EXX= 0.14170570E 07
EYY= 0.14170570E 07
GXY= 0.61891769E 06


               PROPERTIES AT GAGE NO. 8
STRAIN MICRO-IN./IN. ALONG LAMINATE AXIS=-0.550E 02
STRAIN MICRO-IN./IN. 90DEG TO LAMINATE AXIS=-0.139E 03
STRAIN IN MICRO-IN./IN. 45DEG TO LAMINATE AXIS= 0.236E 03
ANGLE OF ROTATION FROM LAMINATE AXIS TO PLANE OF INTEREST IN DEGRESS= 49.000
NORMAL STRESS LB/IN**2 AT PLANE OF INTEREST=-0.604E 03
SHEAR STRESS AT SAME PLANE=-0.118E 03
EXX= 0.14148170E 07
EYY= 0.14148170E 07
GXY= 0.62063406E 06
```