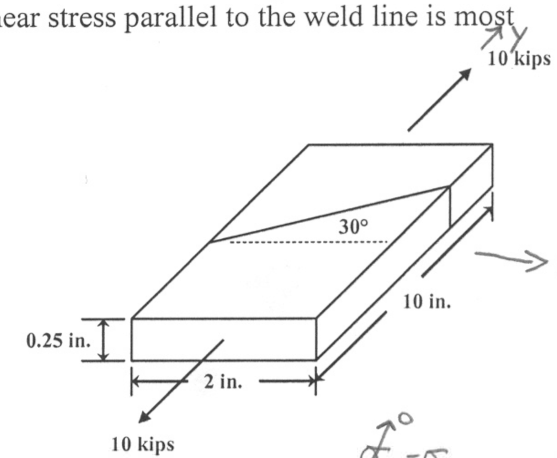
MULTIPLE CHOICE PROBLEMS (10 Points each)

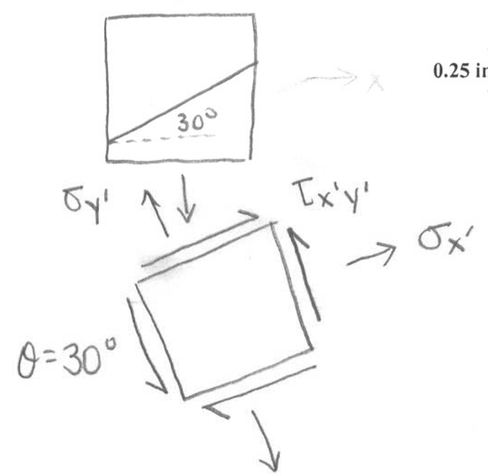
1. A bar is made by welding together two smaller bars as shown. The width of the bar is 2 inches, the length is 10 inches, and the thickness is 0.25 inches. The weld line is at an angle of 30°. If a centric axial force of 10 kips is applied, the magnitude of the shear stress parallel to the weld line is most nearly



$\sigma_x = 0$

$\sigma_y = P/A = \dfrac{10 \text{ kips}}{(2 \text{ in})(0.25 \text{ in})} = 20 \text{ ksi}$

$\tau_{xy} = 0$

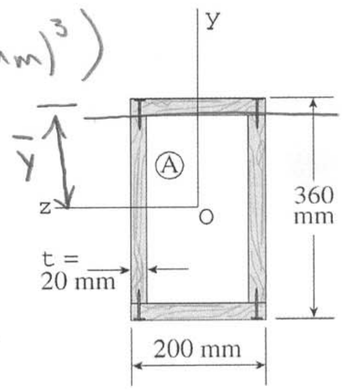


$$\tau_{x'y'} = -\frac{\cancel{\sigma_x}^{\,0} - \sigma_y}{2}\sin 2\theta + \cancel{\tau_{xy}}^{\,0}\cos 2\theta$$

$$= \frac{\sigma_y}{2}\sin(60°)$$

$$= \underline{8.66 \text{ ksi}}$$

2. The box beam shown below is designed for a vertical shear force of 6.4 kN, and is formed by nailing four boards together. If the nails have an allowable shear force of 250 N, the maximum longitudinal nail spacing is most nearly



$$I = \frac{1}{12}\left((200 \text{ mm})(360 \text{ mm})^3 - (160 \text{ mm})(320 \text{ mm})^3\right)$$

$$= 341 \times 10^6 \text{ mm}^4$$

$$Q = A\bar{y} = (200 \text{ mm})(20 \text{ mm})(170 \text{ mm})$$

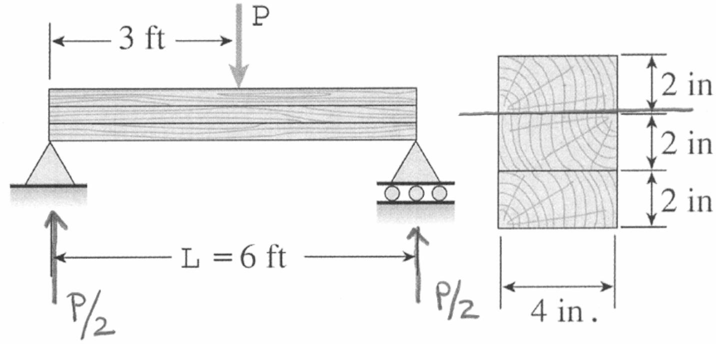
$$= 680 \times 10^3 \text{ mm}^3$$

$$q = \frac{VQ}{I} = 2 \cdot \frac{F_{nail}}{s}$$ (2 nails per location)

$$s = \frac{2F_{nail} I}{VQ} = \frac{2(250 \text{N})(341 \times 10^6 \text{ mm}^4)}{(6400 \text{ N})(680 \times 10^3 \text{ mm}^3)}$$

$$= \underline{39.1 \text{ mm}}$$

3. A laminated wood beam on simple supports is built up by gluing together three 2 in. × 4 in. boards to form a solid beam 4 in. × 6 in. in cross section, as shown in the figure. If the allowable shear stress in the glued joints is 65 psi, the allowable load P is most nearly



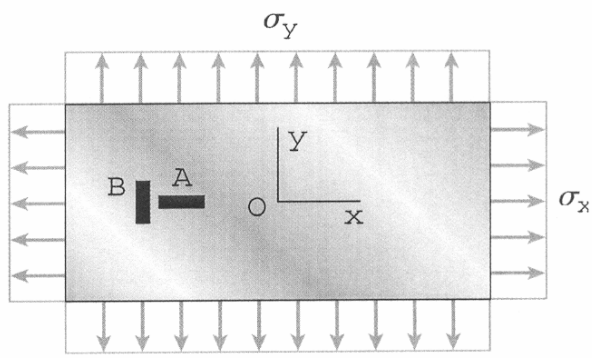
$V = P/2$

$$\tau = \frac{VQ}{It} = \frac{PQ}{2It}$$

$$P_{all} = \frac{\tau_{all} \cdot 2It}{Q}$$

$$= \frac{(65\ \text{lb/in}^2)(2)\left(\frac{1}{12}\right)(4\text{in})(6\text{in})^3\,(4\text{in})}{\underbrace{(2\text{in})(4\text{in})}_{A}\ \underbrace{(2\text{in})}_{\bar{y}}} = \underline{\underline{2340\ \text{lb}}}$$

4. A magnesium plate in biaxial stress is subjected to tensile stresses $\sigma_x = 24$ MPa and $\sigma_y = 12$ MPa (see figure). The corresponding strains in the plate are $\varepsilon_x = 440 \times 10^{-6}$ and $\varepsilon_y = 80 \times 10^{-6}$. The Poisson's ratio of the plate is most nearly



$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right]$$

$$\frac{\varepsilon_x}{\varepsilon_y} = \frac{\sigma_x - \nu\sigma_y}{\sigma_y - \nu\sigma_x}$$

$$\varepsilon_x\sigma_y - \nu\varepsilon_x\sigma_x = \sigma_x\varepsilon_y - \nu\sigma_y\varepsilon_y$$

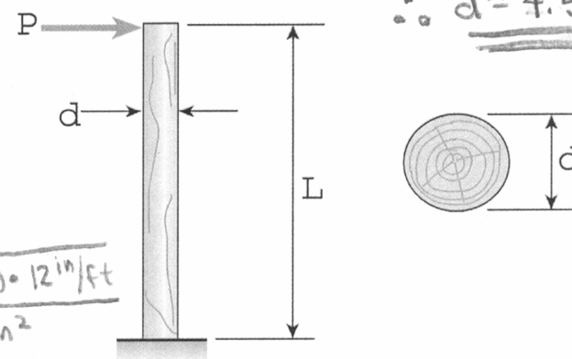
$$\sigma_y\varepsilon_x - \sigma_x\varepsilon_y = \nu\left(\varepsilon_x\sigma_x - \varepsilon_y\sigma_y\right)$$

$$\nu = \frac{\sigma_y\varepsilon_x - \sigma_x\varepsilon_y}{\sigma_x\varepsilon_x - \sigma_y\varepsilon_y} = \frac{(12\text{MPa})(440\times10^{-6}) - (24\text{MPa})(80\times10^{-6})}{(24\text{MPa})(440\times10^{-6}) - (12\text{MPa})(80\times10^{-6})}$$

$$= \underline{\underline{0.35}}$$

5. A wood pole of solid circular cross section (d = diameter) is subjected to a horizontal force P = 250 lb (see figure). The length of the pole is L = 6 ft, and the allowable stresses in the wood are 2000 psi in bending and 150 psi in shear. The required diameter is most nearly

$\therefore d = 4.51\,in$



$$\sigma_{max} = \frac{Mc}{I} = \frac{PL \cdot \frac{d}{2}}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4} = \frac{32}{\pi}\frac{PL}{d^3}$$

$$d = \sqrt[3]{\frac{32}{\pi}\frac{PL}{\sigma_{all}}} = \sqrt[3]{\frac{32}{\pi}\frac{(250\,lb)(6\,ft)\cdot 12\,in/ft}{2000\,lb/in^2}} = 4.51\,in$$

$$\tau_{max} = \frac{4V}{3\pi(d/2)^2} = \frac{16V}{3\pi d^2}$$

$$d = \sqrt{\frac{16V}{3\pi \tau_{all}}} = \sqrt{\frac{16\cdot(250\,lb)}{3\pi\,(150\,lb/in^2)}} = 1.68\,in$$

$\sigma_3 = 0$

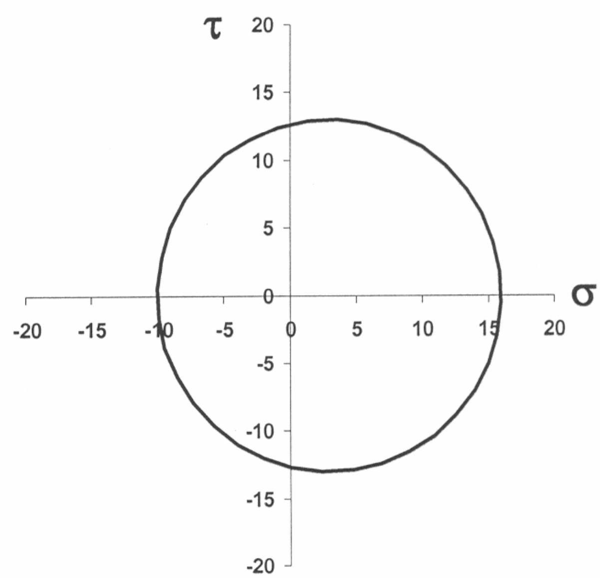
6. The Mohr's circle for the state of plane stress at a point is as shown below. The values of the principal stresses and maximum shear stress are most nearly

$\sigma_1 = 16$

$\sigma_2 = -10$

$\sigma_3 = 0$

$$\tau_{max} = \frac{1}{2}\left|16 - {-10}\right| = 13$$

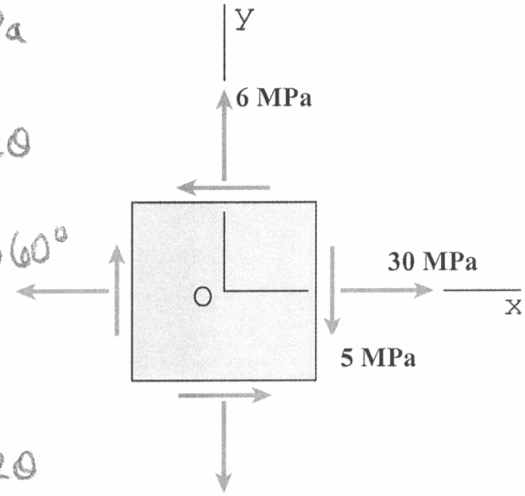


WORK OUT PROBLEM (40 Points)

(a) An element in plane stress is subjected to the stresses shown in the figure at a point O. Determine the stresses on an element oriented at an angle of 30° from the *x* axis, where the angle is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle of 30°.

(b) Determine the principal stresses and show them on a sketch of a properly oriented element.

(c) What is the maximum shear stress at this location?



(a) $\sigma_x = 30\,MPa \quad \sigma_y = 6\,MPa \quad \tau_{xy} = -5\,MPa$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{30+6}{2} + \frac{(30-6)}{2}\cos(60°) + (-5)\sin 60°$$

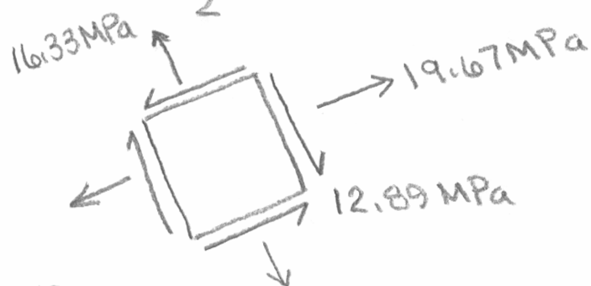
$$= \underline{19.67\,MPa}$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{30+6}{2} - \frac{(30-6)}{2}\cos(60°) - (-5)\sin(60°) = \underline{16.33\,MPa}$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{(30-6)}{2}\sin(60°) + (-5)\cos(60°) = \underline{-12.89\,MPa}$$



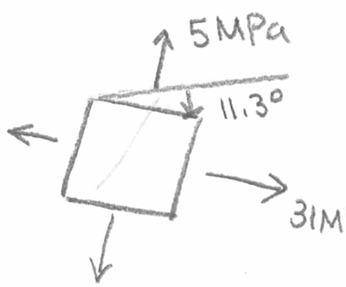
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$= \frac{1}{2}\tan^{-1}\frac{2(-5)}{30-6} = -11.3$$

(b) $$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \frac{30+6}{2} \pm \sqrt{\left(\frac{30-6}{2}\right)^2 + (-5)^2} = 18 \pm 13\,MPa$$

$\sigma_1 = \underline{31\,MPa} \quad \sigma_2 = \underline{5\,MPa} \quad \sigma_3 = 0$



(c) $\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_3| = \underline{15.5\,MPa}$