MULTIPLE CHOICE PROBLEMS (12 Points each)
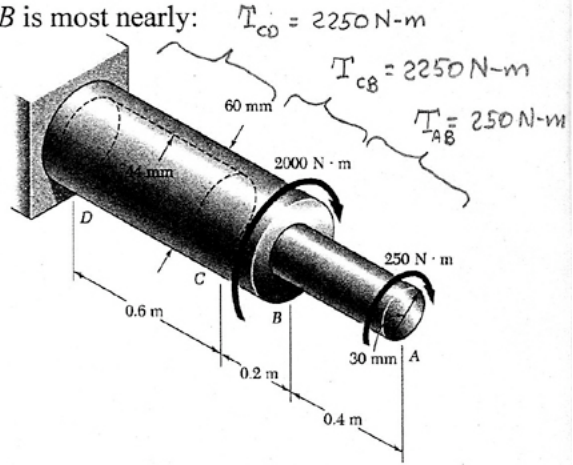
1. The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of aluminum for which $G = 25$ GPa, the angle of twist at B is most nearly:

$T_{CD} = 2250$ N-m

$T_{CB} = 2250$ N-m

$T_{AB} = 250$ N-m

(a) 65.7×10^{-3} rad

(b) 73.9×10^{-3} rad

(c) 8.21×10^{-3} rad

(d) 56.6×10^{-3} rad

(e) 40.3×10^{-3} rad

$$J_{AB} = \frac{\pi (0.015)^4}{2} = 0.07952 \times 10^{-6} \, m^4$$

$$J_{BC} = \frac{\pi (0.03)^4}{2} = 1.272 \times 10^{-6} \, m^4$$

$$J_{CD} = \frac{\pi}{2}\left[(0.3)^4 - (0.022)^4\right]$$
$$[81^{-8} - 23.4^{-8}]$$

$$J_{CD} = 0.905 \times 10^{-6} \, m^4$$

$$\theta_B^@ = \theta_{DC} + \theta_{CB} = \frac{T_{DC} L_{DC}}{J_{DC} G_{DC}} + \frac{T_{CB} L_{CB}}{J_{CB} G_{CB}}$$

$$= \frac{2250 (0.6)}{0.905 \times 10^{-6} (25 \times 10^9)} + \frac{2250 (0.2)}{1.272 \times 10^{-6} (25 \times 10^9)}$$

$$= 59.67 \times 10^{-3} + 14.15 \times 10^{-3}$$

$$\boxed{\theta_B^@ = 73.8 \times 10^{-3} \text{ rad}}$$

60 mm

2000 N · m

250 N · m

0.6 m

0.2 m

0.4 m

30 mm | A

2. Shaft BC is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts AB and CD are solid and of diameter d. If the allowable shearing stress is 75MPa, the required diameter of shaft CD is most nearly

(a) 77.8 mm

(b) 74.1 mm

(c) 37.1 mm

(d) 71.1 mm

(e) 82.2 mm

(CD) solid

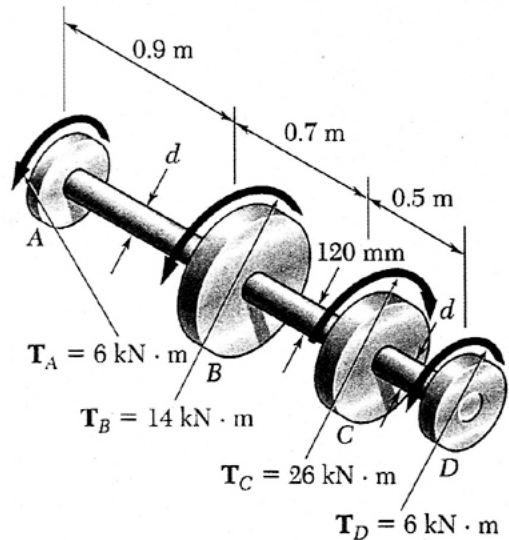$$\tau^{max} = \frac{T c}{J} \quad , \quad J = \frac{\pi c^4}{2}$$

$$\tau^{max} = \frac{2T}{\pi c^3}$$

$$\tau_{CD}^{max} = \frac{2(6000)}{\pi c^3 / 2} = 75 \times 10^6 \, N/m^2$$

Solve for c

$$C = \sqrt[3]{\frac{12,000}{\pi \, 75 \times 10^6}}$$

$$(50.93 \times 10^{-6})$$

$$C = 37.07 \times 10^{-3}$$

$$\boxed{d = 74.14 \times 10^{-3} \, m}$$

0.9 m

0.7 m

0.5 m

d

120 mm

d

$T_A = 6$ kN · m

$T_B = 14$ kN · m

$T_C = 26$ kN · m

$T_D = 6$ kN · m

3. The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 20 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of brass for which the allowable shearing stress is 70 MPa. The largest inner diameter of rod *AB* for which the factor of safety is the same for each rod is most nearly

(a) 20.2 mm
(b) 18.18 mm
(c) 21.1 mm ⟵ (circled)
(d) 17.84 mm
(e) 19.26 mm

D_{AB}^i $C_{BC} = 0.015\,m$

Aluminum $C_{AB}^o = 0.0125\,m$

Brass $C_{AB}^i = ?$

$\tau_{BC}^{all} = 20\,MPa$ B $T = ?$

$\tau_{AB}^{all} = 70\,MPa$ A

$$\tau_{BC}^{max} = \frac{2\,T_{BC}}{\pi\,C_{BC}^3}$$

$$\tau_{AB}^{max} = \frac{2\,T_{AB}\,C_{AB}^o}{\pi\left[\left(C_{AB}^o\right)^4 - \left(C_{AB}^i\right)^4\right]}$$

$$\frac{\tau_{BC}^{max}}{\tau_{BC}^{all}} = F.S._{BC} = F.S._{AB} = \frac{\tau_{AB}^{max}}{\tau_{AB}^{all}}$$

$$\left[(0.0125)^4 - \left(C_{AB}^i\right)^4\right](70) = (0.0125)\,20\,(0.015)^3$$

$$\frac{2T}{(20)\,\pi\,(0.015)^3} = \frac{2T\,(0.0125)}{\pi\left[(0.0125)^4 - \left(C_{AB}^i\right)^4\right](70)}$$

$$\left(C_{AB}^i\right)^4 = (0.0125)^4 - \frac{20}{70}(0.0125)(0.015)^3$$

$$= 24.41 \times 10^{-9} - 12.05 \times 10^{-9}$$

$$= 12.36 \times 10^{-9}$$

$$C_{AB}^i = 10.54 \times 10^{-3}\,m \qquad \boxed{D_{AB}^i = 21.09\,m}$$

4. Knowing that the allowable stress in section *ABD* is 15 ksi, the largest force **P** that can be applied to the bracket shown is most nearly

(a) 934 lb. ⟵ (circled)
(b) 1246 lb.
(c) 467 lb.
(d) 530 lb.
(e) 779 lb.

P ?

A   D
B
0.9 in.
2 in.
0.6 in.
0.6 in.

$$\sigma_B^{max} = \sigma_D^{max} = -\frac{P}{A} - \frac{M_x\,y}{I_x} \qquad y = 0.45$$

$$-15 \times 10^3\,\frac{lb}{in^2} = -\frac{P}{1.08} - \frac{[P(2.45)](0.45)}{72.9 \times 10^{-3}}$$

Solve for P

$$P = 15 \times 10^3 / \left[0.926 + 15.12\right]$$

$$\underline{16.05}$$

$$\boxed{P = 934.6\,lbs}$$

$M_x = P(2.45")$

A   D
0.45   B

$A = 1.08\,in^2$

$$I_x = \frac{bh^3}{12} = 729 \times 10^{-3}\,in^4$$
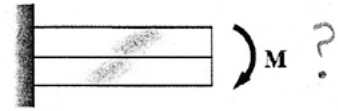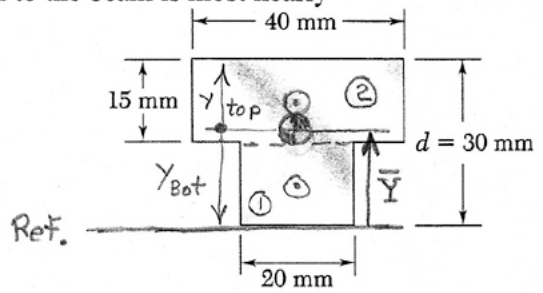
5. The beam shown is made of a nylon for which the allowable stress is 30 MPa in tension and 24 MPa in compression. The largest couple M that can be applied to the beam is most nearly

(a) 148.5 N·m
(b) 84.9 N·m
(c) 141.4 N·m
(d) 176.8 N·m
(e) 173.3 N·m

$\sigma_{all}^{Ten} = 30 \times 10^6 \, Pa$

$\sigma_{all}^{Comp} = 24 \times 10^6 \, Pa$

$\sigma_{all}^{ten} = \dfrac{M y_{top}}{\bar{I}} = 30 \times 10^6 \quad \dfrac{Tension}{(1)}$

$\sigma_{all}^{comp} = \dfrac{M y_{Bot}}{\bar{I}} = 24 \times 10^6 \quad \dfrac{Compression}{(2)}$

Ref.

)M ?

Tension

$(1) \quad 30 \times 10^6 = \dfrac{M(0.0125)}{61.875 \times 10^{-9}}$

$M = \dfrac{61.875 \times 10^{-9} \, [m^4] \, (30 \times 10^6) \, [N/m^2]}{0.0125 \, [m]}$

$M = 148.5 \, N \cdot m$

Compression

$(2) \quad 24 \times 10^6 = \dfrac{M(0.0175)}{61.875 \times 10^{-9}}$

$M = \dfrac{61.875 \times 10^{-9} (24 \times 10^6)}{0.0175}$

$\boxed{M = 84.857 \, N \cdot m}$

lowest Moment
satisfies both Tension
and compression

Region	A mm²	Y mm	yA mm³
①	300	7.5	2250
②	600	22.5	13500
	900	—	15750

$\dfrac{15750}{900} = 17.5 = \bar{Y}$ mm

$y_{top} = 30 - 17.5, \quad y_{top} = 0.0125 \, m$

$y_{Bot} = \bar{Y}, \quad y_{Bot} = 0.0175 \, m$

$d_1 = \bar{Y} - 7.5 = 10 \, mm \quad d_1 = 0.01 \, m$

$d_2 = y_{top} - 7.5 = 5 \, mm \quad d_2 = 0.005 \, m$

$\bar{I}^{①} = \dfrac{b_1 h_1^3}{12} + A_1(d_1)^2 = \dfrac{(0.02)(0.015)^3}{12} + 0.3 \times 10^{-3}(0.01)^2$

$\bar{I}^{①} = 35.625 \times 10^{-9} m^4 = 5.625 \times 10^{-9} + 30.0 \times 10^{-9}$

$\bar{I}^{②} = \dfrac{b_2 h_2^3}{12} + A_2(d_2)^2 = \dfrac{(0.04)(0.015)^3}{12} + 0.6 \times 10^{-3}(0.005)^2$

$\bar{I}^{②} = 26.25 \times 10^{-9} m^4 = 11.25 \times 10^{-9} + 15.0 \times 10^{-9}$

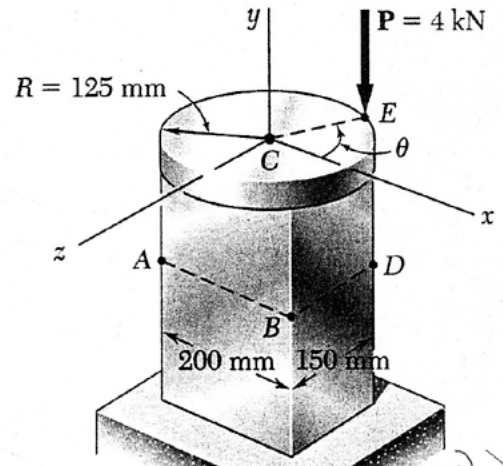$\bar{I} = \bar{I}^{①} + \bar{I}^{②} = 61.875 \times 10^{-9} m = \bar{I}$

problem (4.140)
numbers changed

6. WORK OUT PROBLEM (40 Points)   Name: _Grade Key: R. Kriz_

A rigid circular plate of 125-mm radius is attached to a solid 150x200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with θ = 60°, determine:

(a) the stress at point A
(b) the stress at point B
(c) the point where the neutral axis intersects line *ABD*

y $P = 4$ kN

$R = 125$ mm

E

C θ

x

z A D

B

200 mm 150 mm

(a)

$$\sigma_A = -\frac{P}{A} + \frac{M_x(0.075)}{\bar{I}_x} + \frac{M_z(0.1)}{\bar{I}_z}$$

$$= -\frac{4000}{0.03} + \frac{432(0.075)}{0.05625\times10^{-3}} + \frac{250(0.1)}{0.1\times10^{-3}}$$

$$= -133,333 + 576,000 + 250,000$$

$$\boxed{\sigma_A = 692,667\ Pa}\quad \boxed{10/40}$$

(b)

$$\sigma_B = -\frac{P}{A} + \frac{M_x(0.075)}{\bar{I}_x} - \frac{M_z(0.1)}{\bar{I}_z}$$

$$= -133,333 + 576,000 - 250,000$$
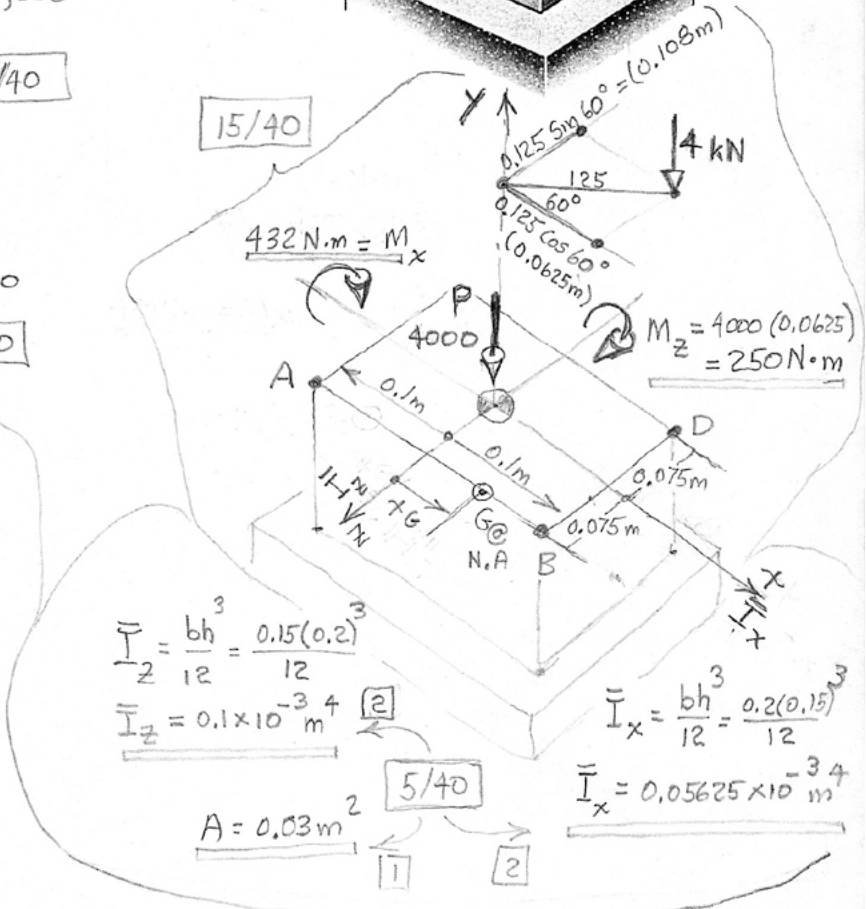
$$\boxed{\sigma_B = 192,667\ Pa}\quad \boxed{10/40}$$

(c) extra credit $\boxed{+5}$: N.A. $\overline{ABD}$

$$\sigma_D = -\frac{P}{A} - \frac{M_x(0.075)}{\bar{I}_x} - \frac{M_z(0.1)}{\bar{I}_z}$$

$$\sigma_D = -133,333 - 576,000 - 250,000$$

$$\sigma_D = -959,333\ Pa$$

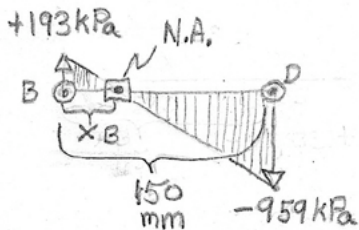Since σ_D is (−) & σ_B is (+)
Then N.A. exists between B & D

$\boxed{15/40}$

$0.125\sin60° = (0.108m)$

4 kN

125
$60°$
$0.125\cos60°$
$(0.0625m)$

$432\ N\cdot m = M_x$

$M_z = 4000(0.0625)$
$= 250\ N\cdot m$

P
4000

A $0.1m$ $0.1m$ D

$0.075m$

x_G $G @$
N.A   B 0.075 m

$\bar{I}_z = \frac{bh^3}{12} = \frac{0.15(0.2)^3}{12}$

$\bar{I}_z = 0.1\times10^{-3}\ m^4$ $\boxed{2}$

$\bar{I}_x = \frac{bh^3}{12} = \frac{0.2(0.15)^3}{12}$

$\bar{I}_x = 0.05625\times10^{-3}\ m^4$

$\boxed{5/40}$

$A = 0.03\ m^2$

$\boxed{1}$ $\boxed{2}$

+193 kPa   N.A.

B   D

x_B

150 mm -959 kPa

$\frac{193}{x_B} = \frac{959}{(150-x_B)}$

$193(150) - 193x_B = 959x_B$

$193(150)/1152 = x_B$

$\boxed{x_B = 25.1\ mm}\ \boxed{+5}$