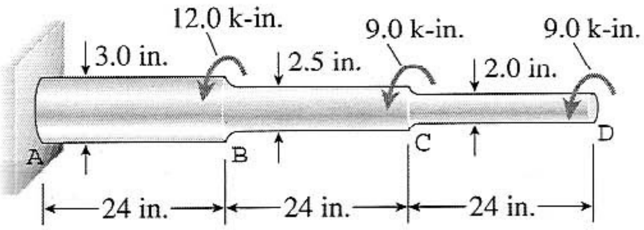
MULTIPLE CHOICE PROBLEMS (10 Points each)

1. A stepped shaft *ABCD* consisting of solid circular segments is subjected to three torques as shown in the Figure. The maximum shear stress, τ_{max}, in the shaft is most nearly:

(a) 5.66 ksi
(b) 5.87 ksi
(c) 733 psi
(d) 5.73 ksi
(e) 2.93 ksi

$$\tau^{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi c^4}{2}}$$

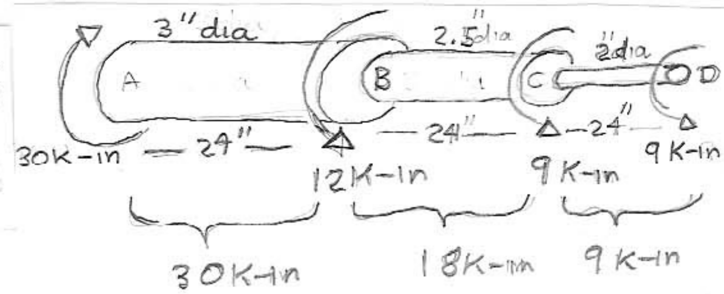
$$\tau^{max} = \frac{2T}{\pi c^3}$$



$$\tau^{max}_{AB} = \frac{2(30{,}000)}{\pi(1.5)^3} \; ; \quad \tau^{max}_{BC} = \frac{2(18{,}000)}{\pi(1.25)^3} \; ;$$

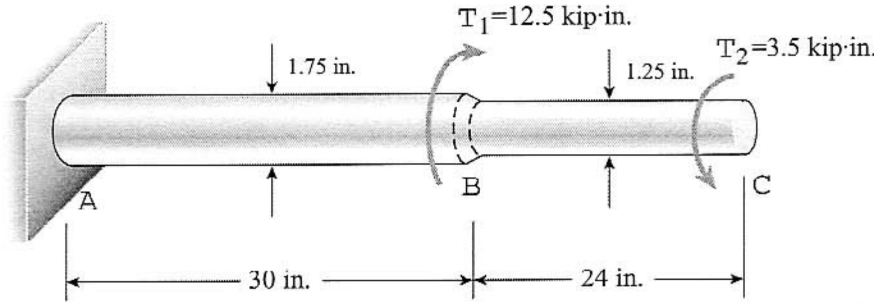
$$\tau^{max}_{AB} = 5.66\ Ksi \; ; \quad \boxed{\tau^{max}_{BC} = 5.87\ Ksi} \; ;$$

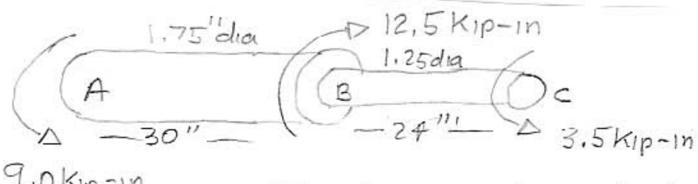
$$\tau^{max}_{CD} = \frac{2(9{,}000)}{\pi(1)^3} \; ; \quad \tau^{max}_{CD} = 5.73\ Ksi$$



2. For the solid brass shaft shown, G = 17x10^3 ksi. The angle of rotation at section B is most nearly:

(a) 1.079 x 10^{-3} radians
(b) 20.6 x 10^{-3} radians
(c) 17.25 x 10^{-3} radians
(d) 24.0 x10^{-3} radians
(e) 73.7 x10^{-3} radians





$$\Theta_{AB} = \frac{T_{AB} L_{AB}}{J_{AB} G} = \frac{9{,}000\,(30)}{\frac{\pi(0.875)^4}{2} \cdot 17\times10^6} = 0.01725 \text{ radians}$$

$$\underline{17.25 \times 10^{-3} \text{ radians}}$$

3. The three forces shown are applied to a rigid plate supported by a solid steel post of radius a. What is the maximum compressive stress in the post?

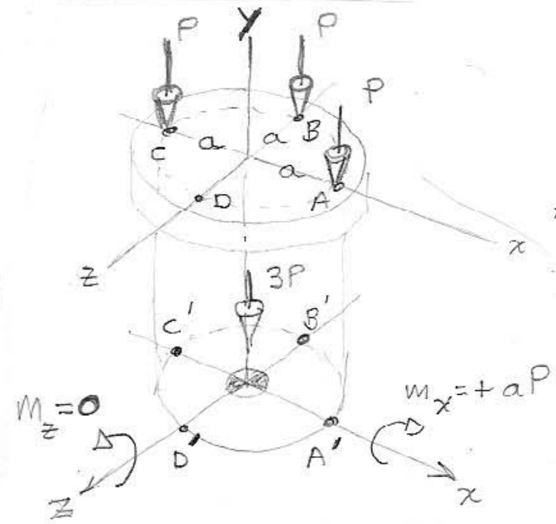
(a) $+\dfrac{P}{\pi a^2}$

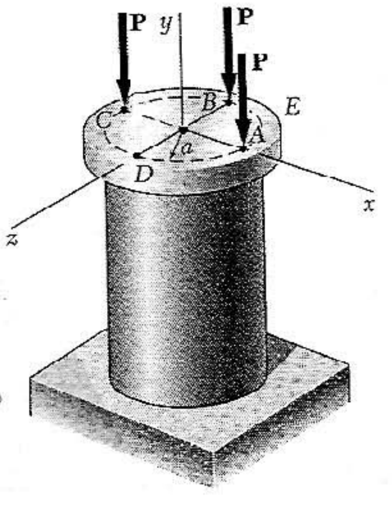
(b) $-\dfrac{P}{\pi a^3}$

(c) $-\dfrac{7P}{\pi a^2}$

(d) $+\dfrac{7P}{\pi a^2}$

(e) $-\dfrac{5P}{\pi a^2}$



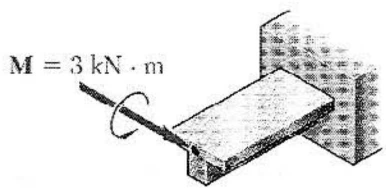


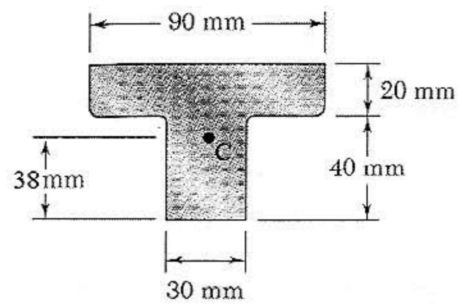
$$\sigma_{B'}^{max} = -\frac{3P}{A} - \frac{M_x c}{I} = -\frac{3P}{\pi a^2} - \frac{4P}{\pi a^2} = \boxed{-\frac{7P}{\pi a^2}}$$

where $c = a,\ A = \pi c^2,\ I = \pi c^4/4$

4. A cast-iron machine part is acted upon by the 3 kN·m couple shown. Knowing that E = 165 GPa, the centroid is located at C, and neglecting the effect of fillets, the maximum compressive stress in the casting is most nearly:

(a) 103.5 MPa
(b) 87.4 MPa
(c) 76.0 MPa
(d) 131.4 MPa
(e) 99.5 MPa





$$\left(\frac{1\ m}{1\times10^3\ mm} \Rightarrow \frac{1\ m^4}{1\times10^{12}\ mm^4}\right)$$

$$\bar{I}_{①} = I_{①} + A d_{①}^2 = \frac{90(20)^3}{12} + (1800)(12)^2 = 60{,}000 + 259{,}200, \qquad y_{top} = 22\ mm$$

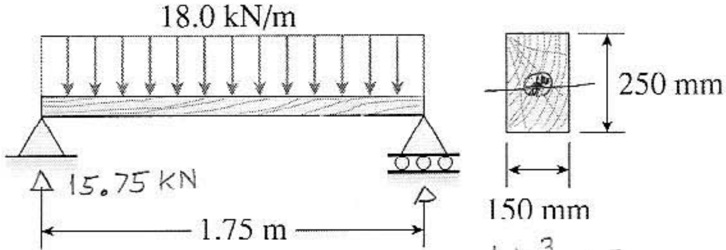
$$\bar{I}_{②} = I_{②} + A d_{②}^2 = \frac{30(40)^3}{12} + (1200)(18)^2 = 160{,}000 + 388{,}800; \qquad y_{Bot} = 38\ mm$$

$$\bar{I} = \bar{I}_{①} + \bar{I}_{②} = 319{,}200 + 548{,}800, \quad \bar{I} = 0.868\times10^6\ mm^4 = 0.868\times10^{-6}\ m^4$$

$$\sigma_{top}^{max} = +\frac{3000\,(0.022)}{0.868\times10^{-6}} = 76.04\ MPa \qquad \sigma_{Bot}^{max} = -\frac{3000\,(0.038)}{0.868\times10^{-6}} = \underline{-131.3\ MPa}$$

5. The maximum bending stress in the simply supported wood beam loaded as shown below is most nearly:

(a) 4.41 MPa
(b) 9.21 MPa
(c) 4.80 MPa
(d) 4.20 MPa
(e) 3.61 MPa



$C = 0.125 m$

$15.75\ kN$

$\frac{bh^3}{12} = I$

$$\sigma^{max} = \frac{M^{max} c}{I} = \frac{6.89\ kN\cdot m\ (0.125 m)}{(0.15 m)(0.25 m)^3/12}$$

$$M^{max} = \frac{15.75\ kN\ (0.875 m)}{2}$$

$$M^{max} = 6.89\ kN\cdot m$$

$$\boxed{\sigma^{max} = 4.41 \times 10^6\ N/m^2}$$

6. Point A is deflected downward by a very small increment, δ, in response to the load, P. The corresponding shear stress in rod *BC* which has a diameter *d* is given by:

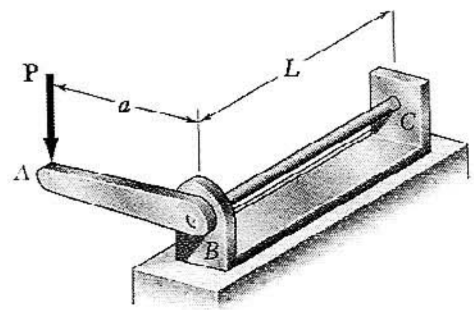
(a) $\tau_{max} = \frac{G\delta a}{Ld}$

(b) $\tau_{max} = \frac{Gd\delta}{La}$

(c) $\tau_{max} = \frac{G\delta d}{2aL}$

(d) $\tau_{max} = \frac{G\delta a}{6Ld}$

(e) $\tau_{max} = \frac{d\delta a}{2GL}$



$$\phi_B = \phi_{B/C} = \delta/a = \frac{TL}{JG} \qquad c = d/2$$

$$T = \frac{\delta}{a}\frac{JG}{L}, \quad \tau_{max} = \frac{Tc}{J} = \frac{\delta/a\ \frac{\cancel{J}G}{L}\left(\frac{d}{2}\right)}{\cancel{J}}$$

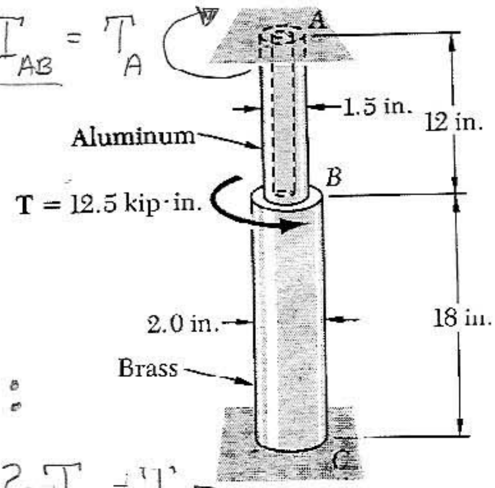
$$\tau_{max} = \frac{\delta}{a}\frac{Gd}{2L}$$

7. WORK OUT PROBLEM (40 Points)

The hollow cylinder *AB* is bonded to the solid cylinder *BC* as shown. The cylinders are attached to rigid supports at *A* and *C*. Knowing that the shear modulus is $4\text{x}10^6$ psi for aluminum, and $6\text{x}10^6$ psi for brass, determine the maximum shearing stress in cylinder *AB*, and the angle of twist at *B*. The inner diameter of the aluminum cylinder is 0.25 in.

Similar to Example 3.05

? $T_{AB} = T_A$



$$\phi = \phi_{AL} + \phi_{Brass} = 0 \quad [\text{Geometry:}]$$

$$+\frac{T_{AB}L_{AB}}{J_{AB}G_{AL}} - \frac{T_{BC}L_{BC}}{J_{BC}G_{Brass}} = 0 \quad (1)$$

Solve equations (1) and (2) for T_{AB} ? & T_{BC} ? :

? $T_{BC} = T_C$

$L_{AB} = 12\text{in.}$ $\quad L_{BC} = 18\text{in.}$

$$J_{AB} = \frac{\pi}{2}\left[(0.75)^4 - (0.125)^4\right]$$

[0.3162] $\quad$ 0.3164 − 0.000244

$J_{AB} = 0.4966\ \text{in}^4$ ✓

$J_{BC} = \pi/2\,(1)^4,\quad J_{BC} = 1.571\ \text{in}^4$ ✓

[Equilibrium:]

$T_A + T_C = 12.5$ kip-in.

$T_{AB} + T_{BC} = 12{,}500$ in-lb $\quad (2)$

Substitute numbers into (1):

$$\frac{T_{AB}\cdot 12}{0.4966\,[4\times10^6]} - \frac{T_{BC}\cdot 18}{1.571\,[6\times10^6]} = 0$$

$6.04\times10^{-6}\,T_{AB} - 1.91\times10^{-6}\,T_{BC} = 0$

$6.04\times10^{-6}(12{,}500 - T_{BC}) - 1.91\times10^{-6}\,T_{BC} = 0$

$T_{AB} = 12{,}500 - T_{BC}$

$-[6.04 + 1.91]\times10^{-6}\,T_{BC} = -0.0755$

$-7.95\times10^{-6}\,T_{BC} = -0.0755$

$T_{BC} = 9497$ in-lb

$T_{AB} = 3003$ in-lb

$$\tau^{max}_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{3003\,(0.75)}{0.4966} = \boxed{4.54\ \text{Ksi}}$$

$$\phi_{BC} = \frac{T_{BC}L_{BC}}{J_{BC}G_{Brass}} = \frac{9497\,(18)}{1.571\,(6\times10^6)} = \boxed{0.0181\ \text{radians} = \phi @_B}$$

$\boxed{1.04^\circ}$