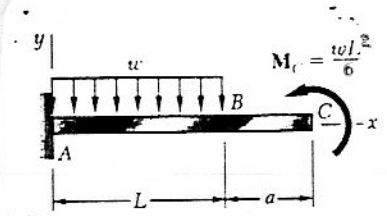
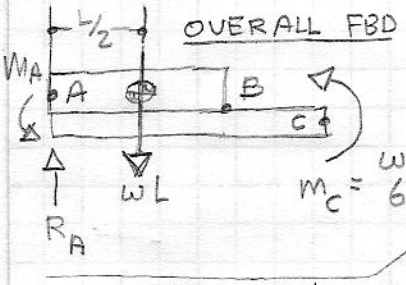
Problem 9.5

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



[1] $x = 0$, $y = 0$

[2] $x = 0$ $dy/dx = 0$

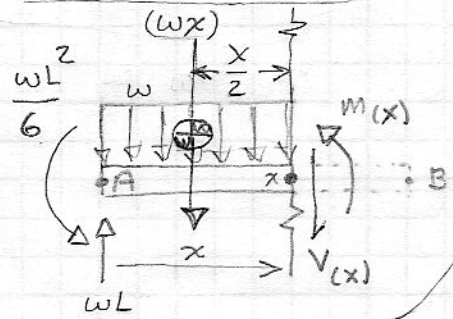


Determine Reactions at A : overall FBD

$$+\uparrow \sum F_y = 0 = R_A - wL \; ; \quad R_A = wL$$

$$\left(+\circlearrowleft\right) \sum M_{@A} = 0 = +M_A - \tfrac{1}{2}(wL) + \frac{wL^2}{6} \; , \quad M_A = +\frac{wL^2}{3}$$

Determine $M_{(x)}$ in interval AB only



$$\left(+\circlearrowleft\right) \sum M_{@x} = 0 = +M_{(x)} + \frac{x}{2}(wx) - wL \cdot x + \frac{wL^2}{3}$$

$$M_{(x)} = -\frac{wx^2}{2} + wLx - \frac{wL^2}{3}$$

Determine $y_{(x)}$ elastic curve in interval AB

$$EI\frac{d^2y_{(x)}}{dx^2} = M_{(x)} = -\frac{wx^2}{2} + wLx - \frac{wL^2}{3}$$

integrate

$$EI\, dy_{(x)}/dx = -\frac{wx^3}{6} + \frac{wLx^2}{2} - \frac{wL^2}{3}x + C_1 = 0$$

Use BC [2], $x = 0$; $dy/dx = 0$ $\Rightarrow C_1 = 0$ (Subst)

$$EI\, dy_{(x)}/dx = -\frac{wx^3}{6} + \frac{wLx^2}{2} - \frac{wL^2x}{3} \quad \text{(1) slope}$$

integrate $\quad EI\, y_{(x)} = -\frac{wx^4}{24} + \frac{wLx^3}{6} - \frac{wL^2x^2}{6} + C_2 = 0$

Use BC [1], $x = 0$; $y = 0$ $\Rightarrow C_2 = 0$ (Subst.)

$$\boxed{y_{(x)} = \frac{w}{24EI}\left(-x^4 + 4Lx^3 - 4L^2x^2\right)}$$ (2) "deformation" elastic curve

Determine deflection at B — Subst. $x = L$ in eqn (2) ; $x = L$,

$$\boxed{y^B_{(x=L)} = -\frac{wL^4}{24EI}}$$ deformation at B

Determine slope at B — Subst. $x = L$ in eqn (1) ; $x = L$,

$$\boxed{\left.\frac{dy}{dx}\right|^B_{x=L} = 0}$$ slope at B