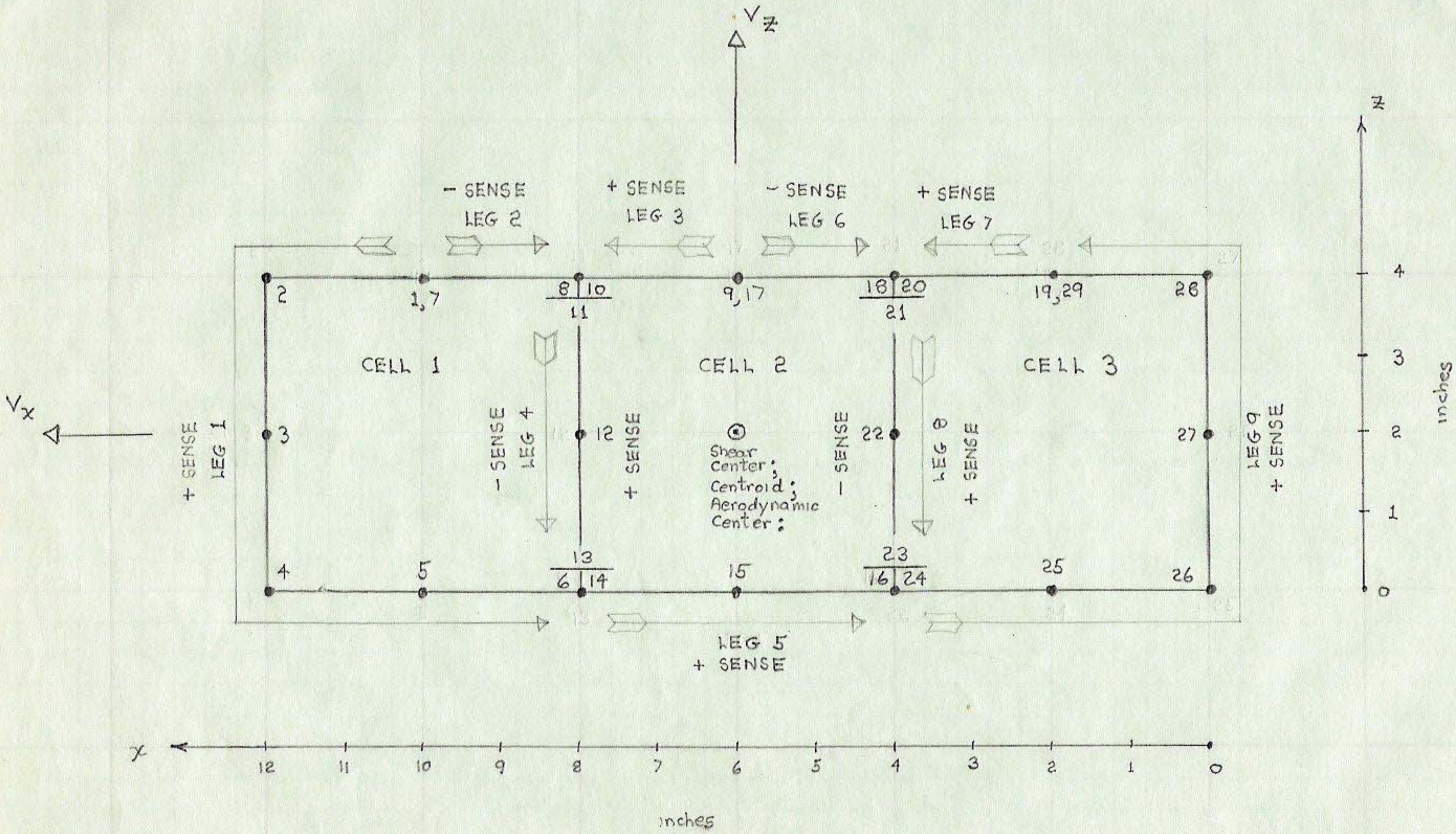
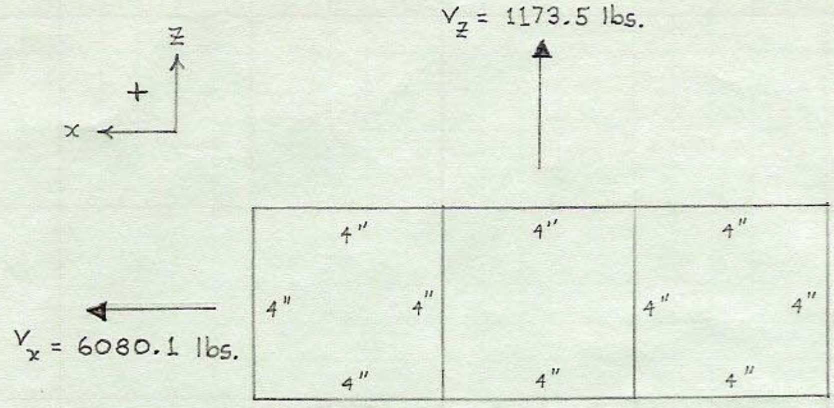


V z
V x
- SENSE
LEG 2
+ SENSE
LEG 3
- SENSE
LEG 6
+ SENSE
LEG 7
2
1,7
8 10
11
9,17
18 20
21
19,29
28
CELL 1
CELL 2
CELL 3
+ SENSE
LEG 1
3
- SENSE
LEG 4
12
+ SENSE
Shear Center;
Centroid;
Aerodynamic Center;
- SENSE
22
LEG 8
+ SENSE
27
LEG 9
+ SENSE
4
5
13
6 14
15
23
16 24
25
26
LEG 5
+ SENSE
x
12
11
10
9
8
7
6
5
4
3
2
1
0
inches
z
4
3
2
1
0
inches



constant
skin thickness
0.1"

Moments of inertia

$$I_x \text{ (For top + Bottom skins)} = \bar{I} + Ad^2 = 2\left\{\frac{12(0.1)^3}{12} + 12(0.1)(2)^2\right\} = 9.602 \text{ in}^4$$

$$I_x \text{ (webs)} = 4\left\{\frac{0.1(4)^3}{12}\right\} = 2.133 \text{ in}^4$$

$$I_x = 11.735 \text{ in}^4$$

$$I_z \text{ (Sides)} = 2\left\{\frac{0.1(12)^3}{12}\right\} = \frac{172.8}{6} = 28.8 \text{ in}^4$$

$$I_z \text{ (Top + Bottom)} = 2\left\{\frac{4(0.1)^3}{12} + 4(0.1)(6)^2\right\} = 28.80067 \text{ in}^4$$

$$I_z \text{ (Top + Bottom interior)} = 2\left\{\frac{4(0.1)^3}{12} + 4(0.1)(2)^2\right\} = 3.20067 \text{ in}^4$$

$$I_z = 60.801 \text{ in}^4$$

Shear Open Cell Calculations
Due to Shear Load V_z

$$q_{i+1} = q_i - \frac{V_z}{I_x}\sum_{n=i}^{i+1} z_{\substack{open \\ i,i+1}} A_{i \to i+1}$$

Leg 1

$q_1 = 0$

$q_{10} = q_1 - 100\{2[2(0.1)]\} = -40$

$q_{20} = q_{10} - 100\{1[2(0.1)]\} = -60$

$q_{30} = q_{20} - 100\{-1[2(0.1)]\} = -40$

$q_{40} = q_{30} - 100\{-2[2(0.1)]\} = 0$

$q_{50} = q_{40} - 100\{-2[2(0.1)]\} = +40$

Stop

Leg 2

$q_{60} = 0$

$q_{70} = q_{60} - 100\{2[2(0.1)]\} = -40$

Stop

Leg 3

$q_{80} = 0$

$q_{90} = q_{80} - 100\{2[2(0.1)]\} = -40$

Leg 4

$q_{100} = q_{70} + q_{90} = -80$

$q_{110} = q_{100} - 100\{1(0.2)\}$

$q_{120} = q_{110} - 100\{-1(0.2)\}$

Stop

Leg 5

$q_{130} = q_{50} + q_{120} = -40$

$q_{140} = q_{130} - 100\{-2(0.2)\} = 0$

$q_{150} = q_{140} - 100\{-2(0.2)\} = +40$

Stop

Leg 6

$q_{160} = 0$

$q_{170} = q_{160} - 100\{2(0.2)\} = -40$

Stop

Leg 7

$q_{180} = 0$

$q_{190} = q_{180} - 100\{2(0.2)\} = -40$

Stop

Leg 8

$q_{200} = q_{170} + q_{190} = -80$

$q_{210} = q_{200} - 100\{1(0.2)\} = -100$

$q_{220} = q_{210} - 100\{-1(0.2)\} = -80$

Stop

Leg 9

$q_{230} = q_{150} + q_{220} = -40$

$q_{240} = q_{230} - 100\{-2(0.2)\} = 0$

$q_{250} = q_{240} - 100\{-2(0.2)\} = +40$

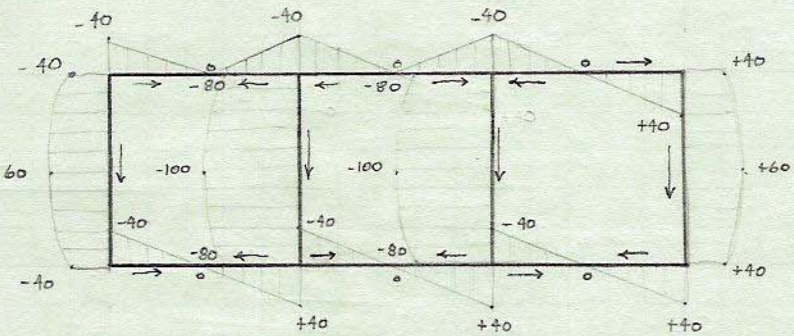
$q_{260} = q_{250} - 100\{-1(0.2)\} = +60$

$q_{270} = q_{260} - 100\{1(0.2)\} = +40$

$q_{280} = q_{270} - 100\{2(0.2)\} = 0$

Stop

V_z Open Cell Shear Flow



Closed Cell Shear Flow Calculations

	Cell 1	Cell 2	Cell 3
$\Sigma q_i L/t_{cell}$	1600	0	-1600
$\Sigma L/t_{cell}$	160	160	160
$\Sigma L/t_{web}$	40	40	
C.O.F.	0.25	0.25, 0.25	0.25
$q_c = -\Sigma q_i L/t / \Sigma L/t_{cell}$	-10	0	+10
C.O.	0	-2.5, +2.5	0
C.O.	0	0, 0	0
$q_c + \Sigma$C.O.	-10	0	+10

$\Sigma q_i L/t_{cell}$

where $q_i = q_i$ (SENSE SIGN)

Cell 1 $\quad \Sigma q_i L/t_1 = \left[-40\frac{4}{0.1} + (-20)\frac{(4)^2/3}{0.1}\right] + \left[80\frac{4}{0.1} + 20\frac{(4)^2/3}{0.1}\right] = 1600$

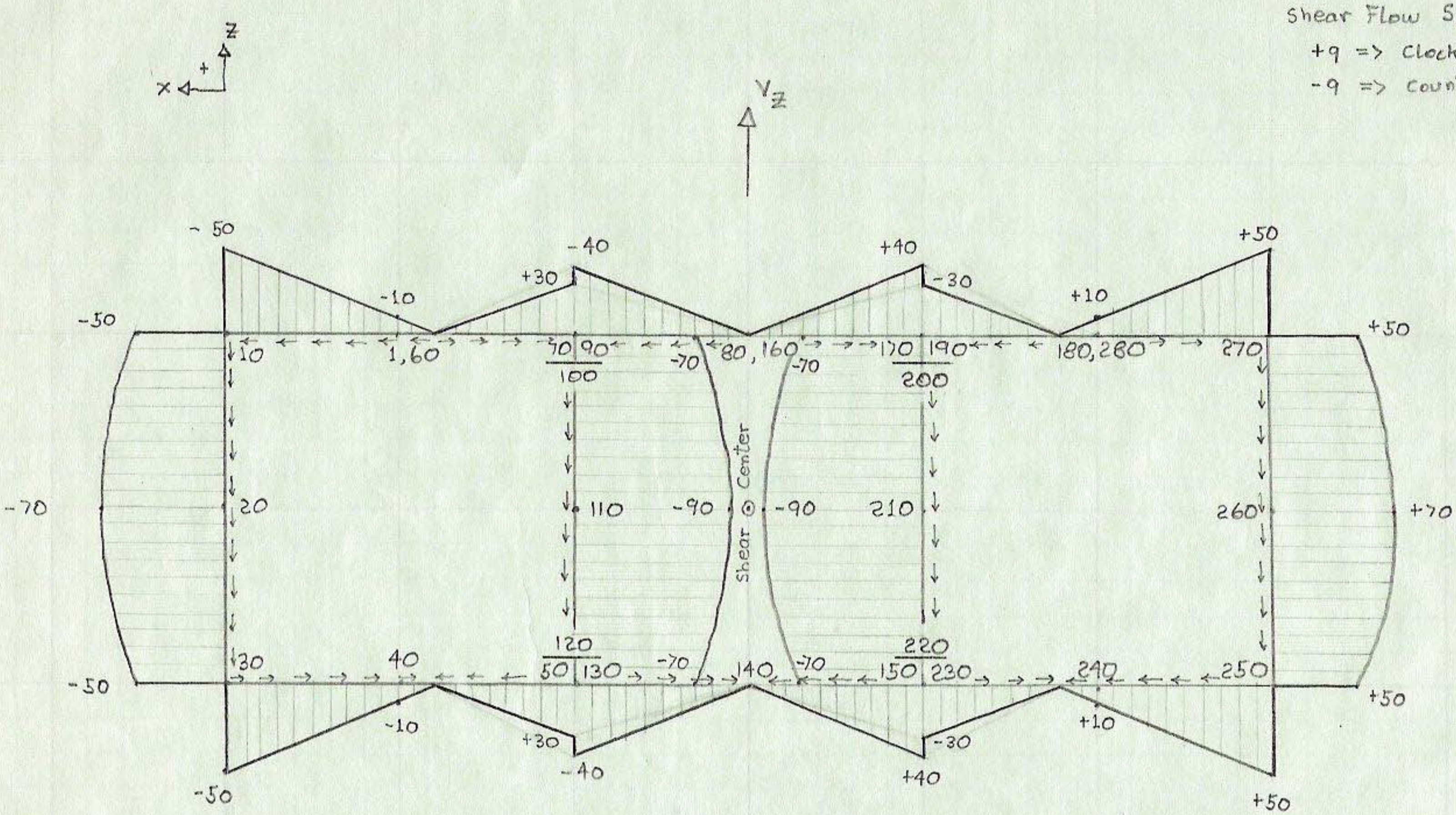
Cell 2 $\quad \Sigma q_i L/t_2 = 0$

Cell 3 $\quad \Sigma q_i L/t_3 = \left[-80\frac{4}{0.1} + (-20)\frac{(4)^2/3}{0.1}\right] + \left[40\frac{4}{0.1} + 20\frac{(4)^2/3}{0.1}\right] = -1600$

Shear Flow Sign Convention

+q => Clockwise

-q => counter Clockwise



Shear Open Cell Calculations
Due to Shear Load V_x

$$q_{i+1} = q_i - \frac{V_x}{I_z} \sum_{n=i}^{i+1} x_{cen} A_{i \to i+1}$$

Leg$_1$

$q_1 = 0$

$q_{10} = q_1 - 100\{5(0.2)\} = -100$

$q_{20} = q_{10} - 100\{6(0.2)\} = -220$

$q_{30} = q_{20} - 100\{6(0.2)\} = -340$

$q_{40} = q_{30} - 100\{5(0.2)\} = -440$

$q_{50} = q_{40} - 100\{3(0.2)\} = -500$

Stop

Leg$_2$

$q_{60} = 0$

$q_{70} = q_{60} - 100\{3(0.2)\} = -60$

Stop

Leg$_3$

$q_{80} = 0$

$q_{90} = q_{80} - 100\{1(0.2)\} = -20$

Stop

Leg$_4$

$q_{100} = q_{70} + q_{90} = -80$

$q_{110} = q_{100} - 100\{2(0.2)\} = -120$

$q_{120} = q_{110} - 100\{2(0.2)\} = -160$

Stop

Leg$_5$

$q_{130} = q_{50} + q_{120} = -660$

$q_{140} = q_{130} - 100\{1(0.2)\} = -680$

$q_{150} = q_{140} - 100\{-1(0.2)\} = -660$

Leg$_6$

$q_{160} = 0$

$q_{170} = q_{160} - 100\{-1(0.2)\} = +20$

Stop

Leg 7

$q_{180} = 0$

$q_{190} = q_{180} - 100\{-3(0.2)\} = +60$

Stop

Leg$_8$

$q_{200} = q_{170} + q_{190} = +80$

$q_{210} = q_{200} - 100\{-2(0.2)\} = +120$

$q_{220} = q_{210} - 100\{-2(0.2)\} = +160$

Stop

Leg$_9$

$q_{230} = q_{150} + q_{220} = -500$

$q_{240} = q_{230} - 100\{-3(0.2)\} = -440$

$q_{250} = q_{240} - 100\{-5(0.2)\} = -340$

$q_{260} = q_{250} - 100\{-6(0.2)\} = -220$

$q_{270} = q_{260} - 100\{-6(0.2)\} = -100$

$q_{280} = q_{270} - 100\{-5(0.2)\} = 0$

Closed Cell Calculations

$\sum q_i L/t_{cell}$ where $q_i = q_i$ (SENSE SIGN)

Cell$_1$ $\sum q_i L/t_1 = \frac{-100(2)2}{3(0.1)} + \frac{-220(4)}{0.1} + \frac{-340(4)}{0.1} + \frac{-160(4)2}{3(0.1)} + \frac{60(2)2}{3(0.1)} + \frac{120(4)}{0.1} = -22400.00$

Cell$_2$ $\sum q_i L/t_2 = \frac{-20(2)2}{3(0.1)} + \frac{-120(4)}{0.1} + \frac{-500(4)}{0.1} + \frac{-180(4)2}{3(0.1)} + \frac{-20(2)2}{3(0.1)} + \frac{-120(4)}{0.1} = -34933.33$

Cell$_3$ $\sum q_i L/t_3 = \frac{60(2)2}{3(0.1)} + \frac{120(4)}{0.1} + \frac{-340(4)}{0.1} + \frac{-160(4)2}{3(0.1)} + \frac{-220(4)}{0.1} + \frac{-100(2)2}{3(0.1)} = -22400.00$

266
3,800

+800.00
+4800.00
+56000.00

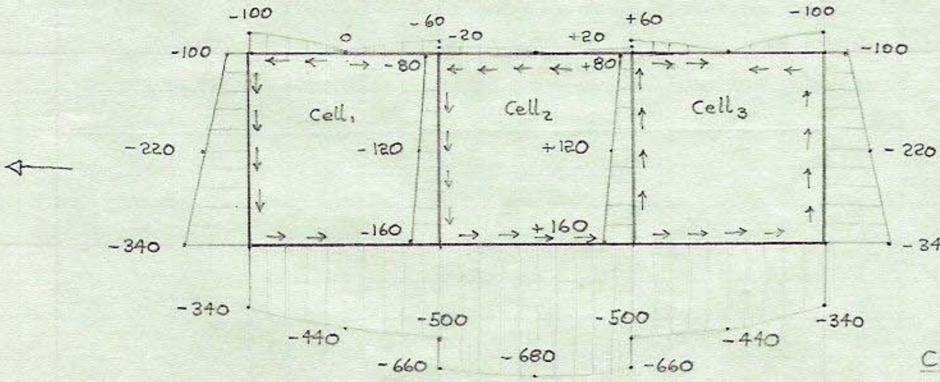
-1333.33
-8800.00
-13600.00
-4266.67
-28000.00
+5600.00
22400.00

140.00
160 | 22400.00

-266.66
-4800.00
20000.00
-4800.00
-266.66
-4800.00
34933.33

218.58
160 | 34933.33
320
293
160
1333
1280
933
800
1333

Shear Flow Open Cell



Closed Cell Shear Flow Calculations

$\Sigma q_i L/t$ cell	-21600.00	-36800.00		-21600.00
$\Sigma L/t$ cell	160	160		160
$\Sigma L/t$ web	40		40	
C.O.F.	0.25	0.25	0.25	0.25
$q_c = -\Sigma q_i L/t / \Sigma L/t_{cell}$	+135.00	+230.00		+135.00
C.O.	57.5	33.75	33.75	57.5
C.O.	16.875	14.375	14.375	16.875
C.O.	7.1875	4.21875	4.21875	7.1875
C.O.	2.1093	1.7968	1.7968	2.1093
C.O.	0.8984	0.52733	.52733	0.8984
C.O.	0.26366	0.2246	.2246	0.26366

C.O.	0.1123	0.065916	0.065916	0.1123
C.O.	0.032958	0.028075	0.0287075	0.032958
C.O.	0.014038	0.0082396	0.0082396	0.014038
$q_c + \Sigma$ C.O.	220.00	340.00		220.00

Displacements For Uniformly Distributed Loads

$$\text{Deflection}_{max} = \frac{WL^4}{8EI} = \frac{81.0706\,(100)^4}{(8)2.5\times10^6(40.5353)} = \frac{2\times10^8}{2\times10^7} = 10.0 \qquad E = 2.5\times10^6\ \text{psi}$$

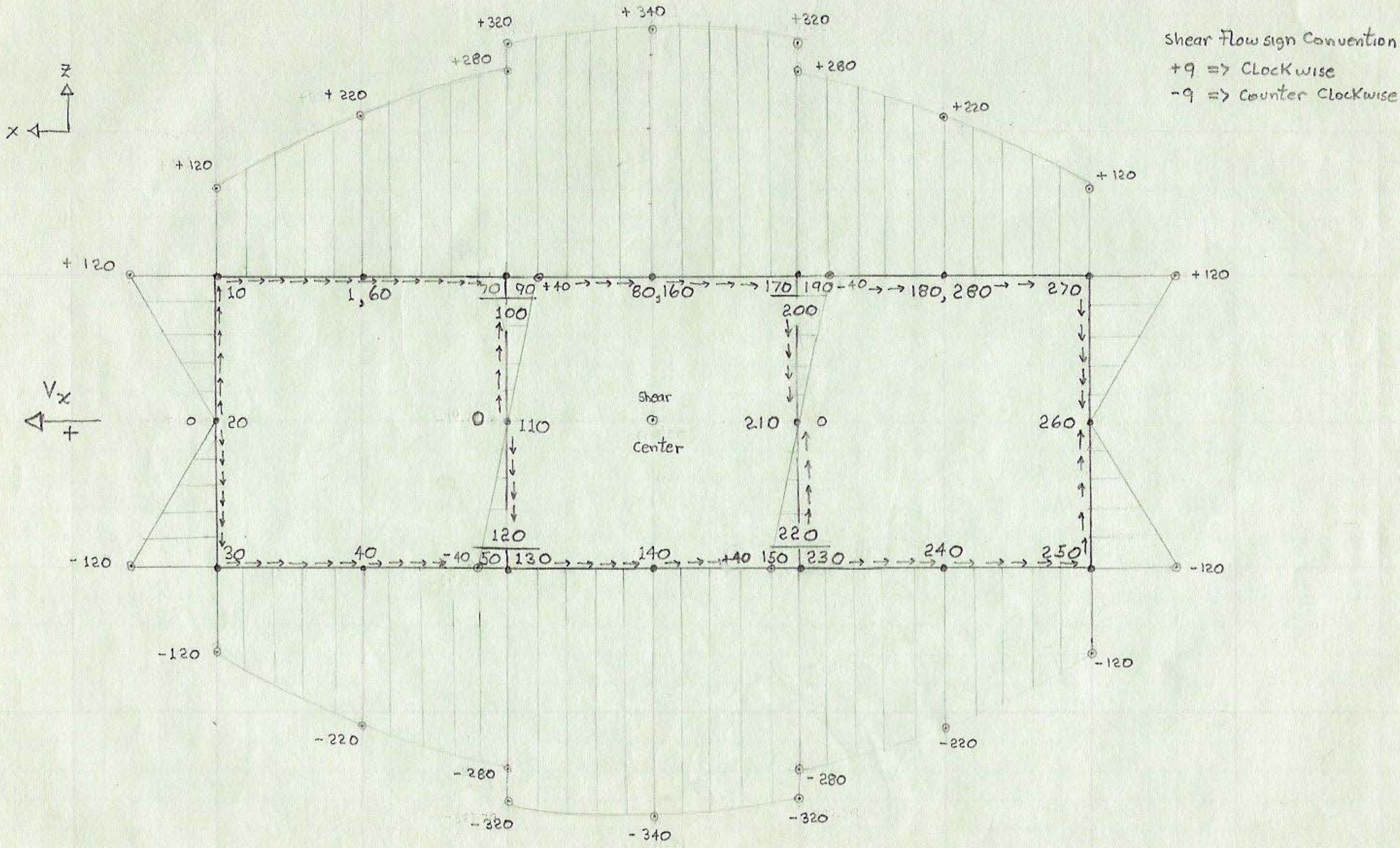
$$\text{Slope}_{max} = \frac{WL^3}{6EI} = \frac{81.0706\,(100)^3}{(6)2.5\times10^6(40.5353)} = \frac{2\times10^6}{15\times10^6} = 0.1333\ \text{radians}$$

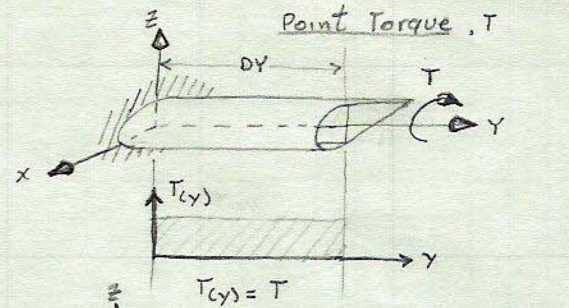
Weight Calculations

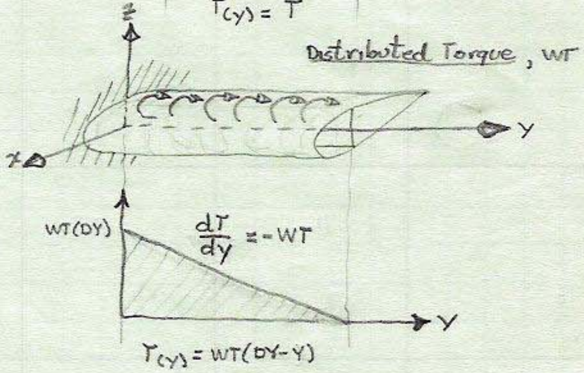
Circumference length = 40 in.
Core thickness = 0.1 in.
Skin thickness = 0.1 in.
Total Volume = 800 $in.^3$
Skin density = 0.025 lb/ $in.^3$
Core density = 0.001 lb/ $in.^3$

Beam Length = 100 in.
Core Volume = 400 $in.^3$
Skin Volume = 400 $in.^3$

Skin Weight = 10 lb.
Core Weight = 0.4 lb.


Shear flow sign Convention
+q => Clockwise
-q => Counter Clockwise
z
x
V_x
+
+120
+220
+280
+320
+340
+320
+280
+220
+120
+120
+120
10
1, 60
70
90
+40
80, 160
170
190
-40
180, 280
270
100
200
0
20
0
110
Shear
Center
210
0
260
120
220
30
40
-40
50
130
140
+40
150
230
240
250
-120
-120
-120
-120
-220
-220
-280
-280
-320
-320
-340





Point Torque, $T_{(y)}$ = constant = T

$$\bar{U} = \int_0^{DY} \oint \frac{q}{2Gt}\, ds\, dy$$

$$q = T/_{2A}$$

$$T_{(y)} = \text{const.}$$

$$U = \int_0^{DY} \frac{T^2}{8A^2} \oint \frac{ds}{Gt}\, dy$$

$$U = \frac{T^2}{8A^2} \oint \frac{ds}{Gt} \int_0^{DY} dy$$

$$\theta_p = \frac{\partial U}{\partial T} = \frac{T}{4A^2G} \oint \frac{ds}{t} \int_0^{DY} dy$$

$$\frac{d\theta_P}{dy} = \frac{T}{4A^2G} \oint \frac{ds}{t} \quad (1) \qquad \text{OR} \qquad \frac{d\theta_P}{dy} = \frac{1}{2AG} \oint \frac{q\,ds}{t} \quad (2)$$

$$\frac{d\theta_P}{dy} = B\ T \quad (3) \qquad \text{where } B = \text{constant} \qquad \theta_p = \left[\frac{1}{2AG} \oint \frac{q\,ds}{t}\right] DY \quad (4)$$

Successive approximation Iteration yields => T, q_1, q_2, ...

Calculate $\frac{d\theta_P}{dy}$ From q_1, q_2, ... using (2)

$$B = \frac{d\theta_P/DY}{T} = \frac{\theta_P/DY}{T} = \frac{\theta_P}{T(DY)}$$

Distributed Torque $\quad T_{(y)} = WT(DY - Y)$

$$\frac{d\theta_D}{dy} = B\ T_{(y)} = B(WT)(DY - Y)$$

$$\theta_D = \int_0^{DY} B(WT)(DY - Y)\, dy$$

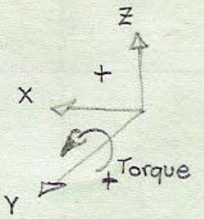
$$= B(WT)\left[(DY)Y - Y^2/_2\right]\Big|_0^{DY}$$

$$= B(WT)\left[(DY)^2 - (DY)^2/_2\right]$$

$$\theta_D = B(WT)\frac{DY^2}{2}$$

$$\theta_D = \frac{\theta_P}{T(DY)}(WT)\frac{DY^2}{2}$$

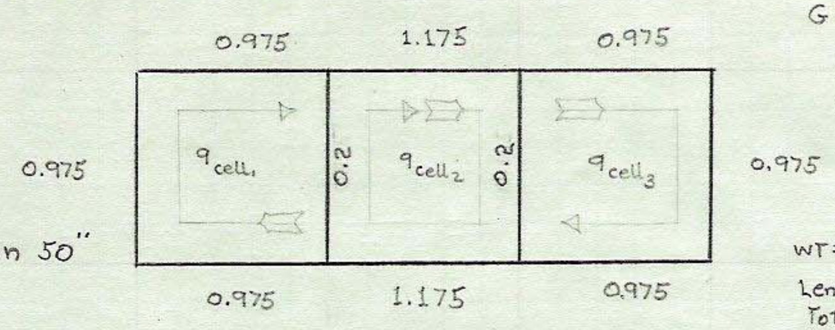
$$\theta_D = \frac{\theta_P(WT)(DY)}{2T}$$



(+ Torque applied = 100 inlb

$G\Theta \equiv 1 \qquad q_{cell_1} = \frac{2A_{cell_1}}{\sum_{cell_1} L/t} = 0.2 \ lb/in = q_{cell_2} = q_{cell_3}$

$G = 3.42 \times 10^5$ psi



Crossection 50"

Shear Flow Sign Convention
+q => Clockwise
−q => Counter-Clockwise

WT = 2 in-lb/in

Length Total = 100 in. = L

2A	32	32	32
$\sum L/t$ cell	160	160	160
$\sum L/t$ web	40	40	
C.O.F	0.25 0.25	0.25 0.25	
$q_c = 2A/\sum L/t$ cell	0.2	0.2	0.2
C.O.	0.05	0.05 0.05	0.05
C.O.	0.025	0.0125 0.0125	0.025
C.O.	0.00625	0.00625 0.00625	0.00625
C.O.	0.003125	0.0015625 0.0015625	0.003125
$q = q_c + \sum C.O.$	0.284375	0.340625	0.284375
2Aq	9.1	10.9	9.1
T_{total} $G\Theta \equiv 1$		29.1	
q corrected	0.975	1.175	0.975

$\frac{T}{29.1} = \frac{100}{29.1} = \frac{q_{cell_1}}{0.2844} = \frac{q_{cell_2}}{0.341} = \frac{q_{cell_3}}{0.2844}$

At Crossection 50" Torque = 100 inlb

$\Theta_p = \left[\frac{1}{2AG} \oint \frac{q\,ds}{t}\right] DY \qquad \Theta_D = \Theta_p \frac{WT(DY)}{2T}$

$\Theta_{T_{0"}} = 0$ (Fixed end)

$\Theta_p = \left[\frac{0.975(12) + (-0.2)4}{2(16)3.42 \times 10^{-5}\, 0.1}\right] 50 \qquad \Theta_D = 0.5 \times 10^{-3} \frac{2(50)}{2(100)}$

$\Theta_p = 0.5 \times 10^{-3}$ radians $\qquad \Theta_D = 0.25 \times 10^{-3}$ radians $\qquad \Theta_{T_{50"}} = 0.75 \times 10^{-3}$ radians

At Crossection 100" Torque = 0

$\Theta_p = 0$ $\qquad \Theta_D$ is the same since the crossectional geometry and distributed torque is the same over DY from 50" to 100"

$\Theta_D = 0.25 \times 10^{-3}$ radians

$\Theta_{T_{100"}} = 0.1 \times 10^{-2}$ radians

Shear Flow for the Combined Shear and Torque loads at Midspan (50" From the root)